4. *Invariant Spherical Distributions of Discrete Series on Real Semisimple Symmetric Spaces G_C/G_R*

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For real semisimple connected Lie groups G_R, Harish-Chandra discussed in [2] invariant eigendistributions on the groups corresponding to the characters of discrete series. In this paper, we study invariant spherical distributions (=ISD's) of discrete series for the symmetric spaces G_C/G_R and the unitary representations associated to the ISD's, for the complexification G_C of G_R. In [6] and [7, 8], the cases of $SL(2, C)/SL(2, R)$, $Sp(2, C)/Sp(2, R)$ and $GL(n, C)/GL(n, R)$ were treated, where the discrete series appears. In [5] and [9], we discussed general theories for the symmetric spaces G_C/G_R. From these works, we can see that there exists an interesting duality between the series of ISD's on G_C/G_R and those of invariant eigendistributions on G_R in such a way that the discrete series corresponds to the continuous series and vise versa.

§1. Invariant spherical distributions of discrete series for G_C/G_R. Assume that G_R has a simply connected complexification G_C. Let σ be an involutive automorphism of G_C such that $(G_C)^\sigma = G_R$, where $(G_C)^\sigma$ is the set of all fixed points of σ in G_C. Put $X=\{g\sigma(g)^{-1} : g \in G_C\}$, then G_C/G_R and X are isomorphic under $G_C/G_R \ni gG_R \mapsto g\sigma(g)^{-1} \in X$ as G_C-spaces. Let $\mathfrak{g}_R$ be the Lie algebra of G_R and $\mathfrak{g}_C$ its complification.

We assume throughout this paper that the symmetric pair $(\mathfrak{g}_C, \mathfrak{g}_R)$ admits a compact Cartan subspace $\mathfrak{b}$. In this case, there exists the discrete series for X. Any root of $(\mathfrak{g}_C, \mathfrak{b}_C)$ is singular imaginary with respect to $\mathfrak{g}_R$ (cf. [10, p. 509]). Let $\mathfrak{a}_1=\mathfrak{b}, \mathfrak{a}_2, \cdots, \mathfrak{a}_n$ be a maximal set of Cartan subspaces of $(\mathfrak{g}_C, \mathfrak{g}_R)$, not G_R-conjugate each other. Recall that $X \subset G_C$ and put $A_i = Z_X(\mathfrak{a}_i)$ and $W^i = N_{G_R}(A_i)/Z_{G_R}(A_i)$ for $1 \leq i \leq n$. Consider the polynomial in t: $\det((1+t)\,\mathrm{Id}\text{-}\mathrm{Ad}(x)) = \sum_{i=0}^m t^i D_i(x)$, $m=\dim \mathfrak{g}_C$. Let l be the smallest integer such that $D_l(x) \not\equiv 0$. The set X' of regular elements in X is an open dense subset of X and $X' = \bigcup_{i=1}^n G_R[A_i']$ with $A_i' = A_i \cap X$ and $G_R[A_i'] = \bigcup_{g \in G_R} gA_i'g^{-1}$. Since $\mathfrak{a}_1$ is compact, the subspace A_1 of X is an abelian connected group. Let A_1^* be the unitary character group of A_1, then it can be identified with a lattice F in the dual space of $\sqrt{-1}\,\mathfrak{b}$: for $\lambda \in F$, there exists a unique element a^* of A_1^* such that $\langle a^*, \exp H\rangle = e^{\lambda(H)}$ $(H \in \mathfrak{b})$. Let W be the Weyl group of $(\mathfrak{g}_C, \mathfrak{b}_C)$. For any $w \in W$, there exists an element $\underline{w} \in W^1$ such that $e^{w\lambda(H)} = \langle a^*, \underline{w}(\exp H)\rangle$ for $H \in \mathfrak{b}$. An element $\lambda \in F$ is called regular if $w\lambda \neq \lambda$ for any $w \in W$, $\neq 1$, and the set of all regular elements of F will be denoted by F'. Denote by $\boldsymbol{D}(X)$ the algebra of G_C-