18. *On the Existence of Periodic Solutions for Periodic Quasilinear Ordinary Differential Systems*

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1. Introduction. In this paper we deal with the problem of the existence of T-periodic solutions for the T-periodic quasilinear ordinary differential system

(1) $$x' = A(t, x)x + F(t, x)$$

where $A(t, x)$ is a real $n \times n$ matrix continuous in (t, x) and T-periodic in t, and $F(t, x)$ is an $\boldsymbol{R}^n$-valued function continuous in (t, x) and T-periodic in t. We consider the associated linear system

(2) $$x' = B(t)x$$

where $B(t)$ is a real $n \times n$ matrix continuous and T-periodic in t.

Hypothesis $\boldsymbol{H}$. There exist no T-periodic solutions of (2) except for the zero solution.

In [1], A. Lasota and Z. Opial discussed the same problem under some hypothesis corresponding to $\boldsymbol{H}$: for each continuous and T-periodic function $y(\cdot)$, $A(\cdot, y(\cdot)) \in M^*$, where M^* is a compact subset of continuous and T-periodic matrices whose systems satisfy Hypothesis $\boldsymbol{H}$. They required that $F(t, x)$ satisfy the following:

$$\liminf_{c \to \infty} \frac{1}{c} \int_0^T \sup_{\|x\| \leqq c} \|F(t, x)\| \, dt = 0.$$

In [2], A. G. Kartsatos considered the existence of T-periodic solutions of (1) under the conditions that $A(t, x)$ is "sufficiently close" to $B(t)$, the system (2) of which satisfies Hypothesis $\boldsymbol{H}$ and that $F(t, x)$ does not depend on x.

In Main Theorem we give an explicit extent that shows how $A(t, x)$ in (1) is close to $B(t)$ in (2) and we obtain certain conditions of $F(t, x)$ which are weaker than those of [1], [2], respectively.

2. Preliminaries. The symbol $\|\cdot\|$ will denote a norm in $\boldsymbol{R}^n$ and the corresponding norm for $n \times n$ matrices. Let C_T be the space of $\boldsymbol{R}^n$-valued functions continuous in $\boldsymbol{R}^1$ and T-periodic with the supremum norm. Let $C[0, T]$ be the space of $\boldsymbol{R}^n$-valued functions continuous on $[0, T]$ with the supremum norm. Let $M[0, T]$ be the space of real $n \times n$ matrices continuous on $[0, T]$ with the supremum norm

$$\|A\|_\infty = \sup\{\|A(t)\|;\ t \in [0, T]\}.$$

We define a bounded linear operator $U: C[0, T] \to \boldsymbol{R}^n$ by $U(x(\cdot)) = x(0) - x(T)$ with the norm

$$\|U\| = \sup\{\|U(x(\cdot))\|;\ \|x\|_\infty = 1\}.$$