R such that $(\alpha-1)\beta=1$. Thus we would have $\alpha=(\alpha^2-\alpha)\beta\in(\alpha^2-\alpha)R$ but $(\alpha^2-\alpha)R\neq R$ because α, and hence $\alpha(\alpha-1)=\alpha^2-\alpha$ is not a unit. Thus α would not be semi-idempotent.

Proposition 5. *Let $R=KG$ be a group ring over an abelian group G. If $\alpha\in R$ is not a zero-divisor and $\alpha-1$ is not a unit in R then α is semi-idempotent.*

Proof. Suppose α be not semi-idempotent. Then $(\alpha^2-\alpha)R$ is a proper ideal of R and $\alpha\in(\alpha^2-\alpha)R$. Thus there is an element $\beta\in R$ such that $\alpha=(\alpha^2-\alpha)\beta=\alpha(\alpha-1)\beta$. As α is not a zero-divisor, we would have $1=(\alpha-1)\beta$, which would mean that $\alpha-1$ is a unit in R.

Note. It is obvious that elements of $R=KG$ of the form kg, $k\,(\neq 0)\in K$, $g\in G$ are units of R. They are called *trivial units*, other units *non-trivial*. It was proved in Passman [2] Chapter 13 that if G is a torsion free abelian group (actually G can be a group of more general type), $R=KG$ has no proper zero-divisors and all units of R are trivial. Using this, we obtain the following theorem, which is the main result of this paper.

Theorem. *Let $R=KG$ be the group ring over a torsion free abelian group G. Let $\alpha\neq 0$ be an element of R which is not a unit. Then α is semi-idempotent if and only if $\alpha-1$ is not a trivial unit.*

Proof. The only-if-part follows from Proposition 4 and the if-part from Proposition 5 and Passman's result.

Remark. The following problems remain open but seem difficult to solve.

(1) Can Proposition 5 be extended into the form: Let K be a field and $R=KG$ the group ring over any group G. If $\alpha-1$ is not a unit in R, then α is semi-idempotent?

(2) Can our Theorem be extended into the form: Let K be a field and $R=KG$ the group ring over any torsion free group G, and suppose $\alpha\,(\neq 0)\in R$ and that α is not a unit. Then α is semi-idempotent if and only if $\alpha-1$ is not of the form kg, $k\in K$, $g\in G$?

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Corrigenda to my former paper in Proc. Japan Acad, **60A**, 333–334 (1984).

p. 333 line 11 from bottom, add "or" between "$1<i$" and "$1<j$".

p. 334 line 7 from above, add "$p\geq$" before "$k\geq 2$".

p. 334 line 10 from bottom, read "$e=\alpha\cdot 1$, $\alpha^2=\alpha\in R$" instead of "$e=0$ or $e=1$".

References

[1] Gray, M.: A Radical Approach to Algebra. Addison Wesley (1970).

[2] Passman, D. S.: The Algebraic Structure of Group Rings. Wiley-Interscience, New York (1977).