91. *On the Algebra of Absolutely Convergent Disk Polynomial Series*

By Yûichi KANJIN
Department of Mathematics, College of Liberal Arts,
Kanazawa University

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Let $\alpha \geqq 0$ and let m, n be nonnegative integers. *Disk polynomials* $R_{m,n}^{(\alpha)}$ are defined in terms of Jacobi polynomials by

$$R_{m,n}^{(\alpha)}(z)=\begin{cases} R_n^{(\alpha,m-n)}(2r^2-1)\,e^{i(m-n)\theta}\,r^{m-n} & \text{if } m\geqq n,\\ R_m^{(\alpha,n-m)}(2r^2-1)\,e^{i(m-n)\theta}\,r^{n-m} & \text{if } m<n,\end{cases}$$

where $z=re^{i\theta}$ and $R_n^{(\alpha,\beta)}(x)$ is the Jacobi polynomial of degree n and of order (α, β) normalized so that $R_n^{(\alpha,\beta)}(1)=1$. If $\alpha=q-2$, $q=2, 3, 4, \cdots$, then disk polynomials are the spherical functions on the sphere S^{2q-1} considered as the homogeneous space $U(q)/U(q-1)$. Let D and $\bar{D}$ be the open unit disk and the closed unit disk in the complex plane, respectively. Denote by $A^{(\alpha)}$ the space of absolutely convergent disk polynomial series on $\bar{D}$, that is, the space of functions f on $\bar{D}$ such that

$$f(z)=\sum_{m,n=0}^{\infty} a_{m,n}R_{m,n}^{(\alpha)}(z) \quad \text{with} \quad \sum|a_{m,n}|<\infty,$$

and introduce a norm to $A^{(\alpha)}$ by $\|f\|=\sum|a_{m,n}|$.

The purpose of this note is to study the structure of the space $A^{(\alpha)}$. Details will be published elsewhere.

1. Firstly we mention some properties of $R_{m,n}^{(\alpha)}$:

(i) $R_{m,n}^{(\alpha)}(z)$ is a polynomial of degree $m+n$ in x and y where $z=x+iy$.

(ii) $\displaystyle\int_{\bar{D}} R_{m,n}^{(\alpha)}(z)R_{k,l}^{(\alpha)}(\bar{z})\,dm_\alpha(z)=h_{m,n}^{(\alpha)-1}\delta_{mk}\delta_{nl}$,

where $dm_\alpha(z)=\left(\dfrac{\alpha+1}{\pi}\right)(1-x^2-y^2)^\alpha\,dx\,dy$, $h_{m,n}^{(\alpha)}=(m+n+\alpha+1)\Gamma(m+\alpha+1)\Gamma(n+\alpha+1)\{(\alpha+1)\Gamma(\alpha+1)^2\Gamma(m+1)\Gamma(n+1)\}^{-1}$, $\bar{z}=x-iy$ and δ_{mk} is Kronecker's δ.

(iii) $|R_{m,n}^{(\alpha)}(z)|\leqq 1$ on $\bar{D}$ ([7; (5.1)]).

(iv) $R_{m,n}^{(\alpha)}(z)R_{k,l}^{(\alpha)}(z)=\sum_{p,q}c_{p,q}(m, n; k, l)h_{p,q}^{(\alpha)}R_{p,q}^{(\alpha)}(z)$ with $c_{p,q}(m, n; k, l)\geqq 0$ ([8; Corollary 5.2]).

Disk polynomials are studied by several authors and we cite here only T. H. Koornwinder [7].

The space $A^{(\alpha)}$ consists of continuous functions on $\bar{D}$ since if $\sum|a_{m,n}|<\infty$ then the series $\sum a_{m,n}R_{m,n}^{(\alpha)}(z)$ converges uniformly on $\bar{D}$ by (iii). Let l^1 be the Banach space of absolutely convergent double sequences $b=\{b_{m,n}\}_{m,n=0}^{\infty}$ with norm $\|b\|=\sum|b_{m,n}|$. Then $A^{(\alpha)}$ is a