# 117. Equivariant Annulus Theorem for 3-Manifolds

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1. **Introduction.** Let $f:(F, \partial F) \to (M, \partial M)$ be a proper map from a bounded surface $F$ into a 3-manifold $M$. The map $f$ is called *boundary incompressible* if it is not properly homotopic to a map $g$: $(F, \partial F) \to (M, \partial M)$ such that $g(F) \subset \partial M$. Let $F'$ be a surface properly embedded in $M$. $F'$ is called *essential* if it is incompressible and inc: $(F, \partial F) \to (M, \partial M)$ is boundary incompressible.

In this paper we will prove an equivariant essential annulus theorem for the Haken manifolds whose boundary components are all tori.

Theorem 1.*) *Let $M$ be a bounded, Haken manifold whose boundary components are all tori and which is not homeomorphic to $T^2 \times I$ where $T^2$ denotes the 2-dimensional torus and $I$ denotes the unit interval* $[0,1]$. *Suppose that there is an essential annulus $A'$ in $M$. If $G$ is a finite subgroup of* Diff $(M)$ *then there exists an essential annulus $A^*$ in $M$ such that either $g(A^*) = A^*$ or $g(A^*) \cap A^* = \phi$ for each element $g$ of $G$.*

Note that for $T^2 \times I$ this theorem does not hold. See the remark of section 2 below.

The examples of 3-manifold admitting no nontrivial, finite group actions are constructed by Raymond-Tollefson [7] and Siebenmann [9]. As an application of Theorem 1 we will give a simple construction of such 3-manifolds by using the knot theory (Theorem 2).

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2. **Proof of Theorem 1.** Throughout this paper we will work in the $C^\infty$-category. For the definitions of standard terms in the three dimensional topology we refer to [3] and [4].

The proof of Theorem 1 depends on the following result which is due to Nakauchi [6]. The author was informed that J. Hass had proved a similar result in his Ph. D. thesis.

Theorem A (Nakauchi). *Let $M$ be a compact, orientable, 3-dimensional, Riemannian manifold with convex incompressible boundary and let $A$ be a smooth annulus. Suppose that there is an essential smooth map $f:(A, \partial A) \to (M, \partial M)$. Then*

(1) *there exists an essential smooth immersion* $f^*:(A, \partial A)$

*) T. Soma independently obtained the similar result.