94. *Generic Bifurcations of Varieties**)

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Let $f:(\boldsymbol{R}^n\times\boldsymbol{R}^r,0)\to(\boldsymbol{R}^p,0)$ be a smooth map germ. For each $u\in(\boldsymbol{R}^r,0)$, we have a germ of "varieties" $f_u^{-1}(0)$ defined by $f_u=f\,|\,\boldsymbol{R}^r\times u$. In this note, we shall announce some results about bifurcations of $f_u^{-1}(0)$ as u varies in $(\boldsymbol{R}^r,0)$. Details will appear elsewhere.

1. Parametrised contact equivalence. The local ring $C_0^\infty(\boldsymbol{R}^n\times\boldsymbol{R}^r)$ is the ring of smooth function germs $(\boldsymbol{R}^n\times\boldsymbol{R}^r,0)\to\boldsymbol{R}$. This ring has a maximal ideal $\mathfrak{M}_{n+r}$ consisting of all germs with $f(0)=0$. For a smooth map germ $f:(\boldsymbol{R}^n\times\boldsymbol{R}^r,0)\to(\boldsymbol{R}^p,0)$, we denote $I(f)=f^*(\mathfrak{M}_p)C_0^\infty(\boldsymbol{R}^n\times\boldsymbol{R}^r)$, where $f^*:C_0^\infty(\boldsymbol{R}^p)\to C_0^\infty(\boldsymbol{R}^n\times\boldsymbol{R}^r)$ is defined by $f^*(h)=h\circ f$.

Definition 1. Map germs $f,g:(\boldsymbol{R}^n\times\boldsymbol{R}^r,0)\to(\boldsymbol{R}^p,0)$ are *P-$\mathcal{K}$-equivalent* (resp. *S.P-$\mathcal{K}$-equivalent*) if there exists a diffeomorphism germ on $(\boldsymbol{R}^n\times\boldsymbol{R}^r,0)$ of the form $\Phi(x,u)=(\Phi_1(x,u),\phi(u))$ (resp. $\Phi(x,u)=(\Phi_1(x,u),u)$) such that $\Phi^*(I(f))=I(g)$. We denote $f\sim_{P\text{-}\mathcal{K}}g$ (resp. $f\sim_{S.P\text{-}\mathcal{K}}g$).

For each smooth map germ $f:(\boldsymbol{R}^n\times\boldsymbol{R}^r,0)\to(\boldsymbol{R}^p,0)$, the *bifurcation map germ* $\pi_f:(f^{-1}(0),0)\to(\boldsymbol{R}^r,0)$ is defined by $\pi_f(x,u)=u$.

Definition 2. For two map germs $f,g:(\boldsymbol{R}^n\times\boldsymbol{R}^r,0)\to(\boldsymbol{R}^p,0)$, bifurcation map germs π_f,π_g are *$\mathcal{A}$-equivalent* if there are diffeomorphism germs Φ on $(\boldsymbol{R}^n\times\boldsymbol{R}^r,0)$ and ϕ on $(\boldsymbol{R}^r,0)$ such that $\Phi(f^{-1}(0))=g^{-1}(0)$ and $\phi\circ\pi_f=\pi_g\circ\Phi$.

Remarks. i) If f,g are P-$\mathcal{K}$-equivalent, then π_f, π_g are $\mathcal{A}$-equivalent.

ii) For each $f:(\boldsymbol{R}^n,0)\to(\boldsymbol{R}^p,0)$, we define $D_f:(\boldsymbol{R}^n\times\boldsymbol{R}^p,0)\to(\boldsymbol{R}^p,0)$ by $D_f(x,y)=f(x)-y$. We can see that P-$\mathcal{K}$-equivalence theory is one of the generalization of Mather's $\mathcal{A}$-equivalence theory (cf. [3], [4]).

iii) The case when $r=1$, this equivalence relation has been studied by M. Golubitsky and D. Schaeffer ([1]). But the situation is quite different in the case of $r\geq 2$ (see the next section).

2. Finite determinacy. Definition 3. Let $f,g:(\boldsymbol{R}^n\times\boldsymbol{R}^r,0)\to(\boldsymbol{R}^p,0)$ be smooth map germs. i) f,g are *k-jet equivalent* if $(f^*-g^*)(\mathfrak{M}_p)\subset\mathfrak{M}_{n+r}^{k+1}$. ii) f,g are *(k_1,k_2)-jet equivalent* if $(f^*-g^*)(\mathfrak{M}_p)\subset(\mathfrak{M}_n^{k_1+1}+\mathfrak{M}_r^{k_2+1})C_0^\infty(\boldsymbol{R}^n\times\boldsymbol{R}^r)$.

These are clearly equivalence relations; we respectively denote j_0^kf and $j_0^{(k_1,k_2)}f$ of equivalence classes represented by f.

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