53. The Structure of Open Algebraic Surfaces and Its Application to Plane Curves

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The purpose of this note is to outline our recent results on the structure of algebraic surfaces which may not be complete. Details will be published elsewhere.

1. A triple $(X, \bar{X}, D)$ is said to be a non-singular triple, if $\bar{X}$ is a complete non-singular surface over the field of complex numbers and if D is a divisor with only simple normal crossings such that $X=\bar{X}\backslash D$. We denote by $K(\bar{X})$ the canonical divisor on $\bar{X}$. We define logarithmic m-genera $\bar{P}_m(X)$ and the logarithmic Kodaira dimension $\bar{\kappa}(X)$ by

$$\bar{P}_m(X)=\dim H^0(\bar{X}, m(K(\bar{X})+D)),$$
$$\bar{\kappa}(X)=\kappa(K(\bar{X})+D, \bar{X})$$

(see [2]).

In general, let Δ be a divisor on $\bar{X}$ with $\kappa(\Delta, \bar{X})\geq 0$. Then one has a $\boldsymbol{Q}$-divisor Δ^+ and an effective $\boldsymbol{Q}$-divisor Δ^- such that

(1) $\Delta=\Delta^+ + \Delta^-$,

(2) Δ^+ is semipositive (i.e. $(\Delta^+, \Gamma)\geq 0$ for all curves Γ on $\bar{X}$),

(3) the intersection matrix of Δ^- is negative-definite or $\Delta^-=0$,

(4) $(\Delta^+, \Delta^-)=0$.

This decomposition is unique and is called the Zariski decomposition of Δ (see [4] or [5]).

The main results are summarized as follows:

Theorem 1. *If $\bar{\kappa}(X)=0$, then $\bar{P}_i(X)=1$ for some i, $1\leq i\leq 66$.*

Theorem 2. *If $\bar{\kappa}(X)\geq 0$ and if D is connected, then $\bar{P}_{12}(X)>0$.*

We shall outline proofs of these theorems. A triple $(X, \bar{X}, D)$ is said to be almost minimal if the support of $(K(\bar{X})+D)^-$ contains no exceptional curve of the 1st kind.

Lemma 3. *Given a triple $(X, \bar{X}, D)$ with $\bar{\kappa}(X)\geq 0$, there exist an almost minimal triple $(Z, \bar{Z}, B)$ and a birational morphism $f\colon \bar{X}\to\bar{Z}$ having the following properties:*

(1) $B=f_*(D)$,

(2) $(K(\bar{X})+D)^+=f^*((K(\bar{Z})+B)^+)$.

By the above lemma, it suffices to prove the theorems for almost minimal triples $(X, \bar{X}, D)$. We need the following

Proposition 4. *If $(X, \bar{X}, D)$ is almost minimal, then $D-(K(\bar{X})+$ $+D)^-)$ is effective and $(\bar{X}, D-(K(\bar{X})+D)^-)$ is a relatively minimal model*