53. *Integration of Correspondences and a Variational Problem with Operator Constraint*[*)]

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(Communicated by Shokichi IYANAGA, M. J. A., May 12, 1980)

1. Introduction. Let $u:[0,1]^2\times \boldsymbol{R}^l_+\to\boldsymbol{R}$ and $\omega:[0,1]\to\boldsymbol{R}^l_+$ be two fixed mappings, and consider the following variational problem with a control variable $x:[0,1]^2\to\boldsymbol{R}^l_+$:

$$(*)\qquad \begin{aligned}&\underset{x}{Maximize}\int_0^1\int_0^1 u(s,t,x(s,t))dsdt\\ &subject\ to\\ &\qquad\int_0^1 x(s,t)ds\leqq\omega(t)\qquad\text{for all } t\in[0,1].\end{aligned}$$

($\boldsymbol{R}^l_+$ designates the nonnegative orthant of $\boldsymbol{R}^l$.)

We can easily give this problem a lot of economic interpretations. For example, $x(s,t)$ can be interpreted as an allocation of various resources among agents $s\in[0,1]$ over time-interval $[0,1]$. The available quantities of these resources at each time $t\in T$ are represented by $\omega(t)$. Then $(*)$ is a formal expression of the problem to maximize the sum of utilities of all agents over time subject to the resource-constraint. (Cf. Maruyama [10] for a related problem.)

In this paper, we are going to establish a set of sufficient conditions which assures the existence of an optimal solution for this kind of variational problem. Several new results on infinite dimensional Ljapunov measures and the integration theory of correspondences (=multivalued mappings) will also be presented as indispensable preparations for our main purpose.

Arkin-Levin [1] examined a similar problem and I am very much indebted to them for various ideas.

2. Abstract integrals of correspondences. Throughout this section, Ω stands for a compact Hausdorff space, $\mathfrak{X}$ and $\mathfrak{Y}$ for Banach spaces, and $C(\Omega,\mathfrak{X})$ for the set of all the continuous mappings $f:\Omega\to\mathfrak{X}$. $C(\Omega,\mathfrak{X})$ is a Banach space whose topology is defined by the sup-norm:

$$\|f\|=\sup_{\omega\in\Omega}\|f(\omega)\|.$$

We designate by $\mathcal{L}(\mathfrak{X},\mathfrak{Y})$ the space of all the bounded linear operators on $\mathfrak{X}$ into $\mathfrak{Y}$. Let

*) The earlier version of this paper was read at the annual meeting of the Japan Association of Economics and Econometrics in 1979. The financial support from Keio Gijuku Academic Promotion Funds is gratefully acknowledged.