$$A_{-}=\frac{1}{2}\begin{pmatrix} -i\xi & -(1+i\xi)e^{-i(x+\chi-\eta)} \\ (1+i\xi)e^{i(x+\chi-\eta)} & 2+i\xi \end{pmatrix}.$$

The fact that (20) has two regular singularities and an irregular singularity of rank one implies that the deformation equation $d\Omega-\Omega^2=0$ is integrable in terms of a Painlevé transcendent of the fifth kind. In our case, substituting (22) to

$$x\frac{dA_{\pm}}{dx}=\left[\begin{pmatrix} 0 & e_{12} \\ e_{21} & \pm ix \end{pmatrix}, A_{\pm}\right] \tag{23}$$

we obtain (7) and

$$x\frac{d\chi}{dx}=2\xi\cos^2\left(\frac{\eta}{2}\right)+2i\sin^2\left(\frac{\eta}{2}\right). \tag{24}$$

Finally (18) yields

$$\begin{aligned}\frac{d}{dx}\log\rho(x)&=i(A_{+}-A_{-})_{22}+\frac{1}{x}e_{12}e_{21}\\ &=\xi+\frac{1}{2x}(1+\xi^2)(1+\cos\eta)-\frac{1}{x},\end{aligned} \tag{25}$$

which is the results (3) and (8).

Erratum and comment for XI [9].

p. 8, (37) [9] should be corrected as $\sigma[M]=\tau[T_M]^2$.

Theorem 5 has previously been obtained by Widom [10]. He has also shown [11] that the product of half infinite Toeplitz operator (32) [9] of M with that of M^{-1} differs from 1 by a trace-class operator, and that $\sigma[M]$ coincides with its determinant. The authors wish to thank Prof. H. Widom for calling their attention to his results.

References

[1] H. G. Vaidya and C. A. Tracy: Phys. Rev. Lett. **42**, 3–6 (1979).
[2] M. Jimbo, T. Miwa, and M. Sato: Proc. Japan Acad. **55**A, 267–272 (1979).
[3] A. Lenard: J. Math. Phys. **5**, 930–943 (1964).
[4] M. Hukuhara, T. Kimura, and T. Matuda: Equations différentielles ordinaires du premier ordre dans le champ complexe. The Mathematical Society of Japan (1961).
[5] H. G. Vaidya and C. A. Tracy: Phys. Lett. **68**A, 378–380 (1978).
[6] T. D. Schultz: J. Math. Phys. **4**, 666–671 (1963).
[7] H. G. Vaidya: Thesis. State Univ. of New York at Stony Brook (1978).
[8] M. Sato, T. Miwa, and M. Jimbo: Publ. RIMS. Kyoto Univ., **15**, 201–278 (1979).
[9] ——: Proc. Japan Acad. **55**A, 6–9 (1979).
[10] H. Widom: Adv. Math. **13**, 284–322 (1974).
[11] ——: Proc. Amer. Math. Soc. **50**, 167–173 (1975); Adv. Math. **21**, 1–29 (1976).