63. Studies on Holonomic Quantum Fields. XV

Double Scaling Limit of One Dimensional XY Model

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The aim of this article is to show that the double scaling limit ([6] [7] [8]) of the one dimensional XY chain can be handled in the framework of monodromy preserving deformation theory (cf. [1] [2] [3]).

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1. The one-dimensional spin $\frac{1}{2}$ XY model is described by the Hamiltonian

$$(1)\qquad H_M=-\frac{1}{4}\sum_{m=0}^{M-1}((1+\gamma)\sigma_m^x\sigma_{m+1}^x+(1-\gamma)\sigma_m^y\sigma_{m+1}^y+2h\sigma_m^z)$$

$$\sigma_m^\alpha=I_2\otimes\cdots\otimes\overset{m}{\breve{\sigma^\alpha}}\otimes\cdots\otimes I_2\qquad(\alpha=x,y,z)$$

where $\sigma^x=\begin{pmatrix} & 1\\ 1 & \end{pmatrix}$, $\sigma^y=\begin{pmatrix} & -i\\ i & \end{pmatrix}$, $\sigma^z=\begin{pmatrix}1 & \\ & -1\end{pmatrix}$.

In the sequel we shall be concerned with the double scaling limit of the model (1), defined as follows ([6]):

$$(2)\qquad m,n\to\infty\,;\ \varepsilon=\sqrt{1-h^2}\to 0,\ \gamma\to 0$$

$$\text{keeping } g=\gamma/\varepsilon>0,\ a=m\varepsilon,\ t=\frac{n}{2}\varepsilon^2 \text{ fixed.}$$

The result is quite similar to the scaling limit of the Ising model, except that the characteristic dispersion relation $\omega(p)=\sqrt{p^2+m^2}$ for the latter is now replaced by ([6])

$$(3)\qquad \omega(p)=\sqrt{(p^2+\mu^2)(p^2+\mu^{*2})}$$

where $\mu=g+\sqrt{g^2-1}$ $(g\geqq 1)$, $=g+i\sqrt{1-g^2}$ $(0<g\leqq 1)$, $\mu^*=\mu^{-1}$. Denote by $\psi^\dagger(p)$, $\psi(p)$ the creation-annihilation operators of free fermion such that $[\psi^\dagger(p),\psi(p')]_+=2\pi\delta(p-p')$, and set

$$(4)\qquad \hat{\psi}_\pm(p,t)=\psi^\dagger(-p)e^{it\omega(p)}\pm\psi(p)e^{-it\omega(p)}$$

$$(5)\qquad \psi^{(\pm,n)}(a,t)=\int_{-\infty}^{+\infty}\frac{dp}{2\pi}\sqrt{\omega(p)}^{\pm1}\hat{\psi}_\pm(p,t)e^{iap}p^n=\left(\frac{1}{i}\frac{\partial}{\partial a}\right)^n\psi^{(\pm)}(a,t)$$

$$(n=0,1,2,\cdots)$$