26. *Tychonoff Functor and Product Spaces*

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1. Introduction. In this paper a space means a topological space with no separation axiom unless otherwise specified. We use the term "Tychonoff functor" in the sense of K. Morita [2] and denote it by τ which is the epi-reflective functor from the category of all spaces and continuous maps onto the category of all Tychonoff spaces and continuous maps.

For any spaces X and Y, we denote by $f_{X,Y}$ the unique continuous map from $\tau(X\times Y)$ onto $\tau(X)\times\tau(Y)$ which makes the following diagram commutative, where the symbol Φ_X follows [2].

$$\begin{tikzcd} X\times Y \arrow[rr, "\Phi_X\times\Phi_Y"] \arrow[dr, "\Phi_{X\times Y}"'] & & \tau(X)\times\tau(Y) \\ & \tau(X\times Y) \arrow[ur, "f_{X,Y}"'] & \end{tikzcd}$$

The equality $\tau(X\times Y)=\tau(X)\times\tau(Y)$ means that $f_{X,Y}$ is a homeomorphism. Concerning this equality, the following theorems are known.

Theorem 1 (K. Morita). *$\tau(X\times Y)=\tau(X)\times\tau(Y)$ is valid if and only if every cozero set of $X\times Y$ can be expressed as the union of rectangular cozero sets of $X\times Y$.*

A subset V of $X\times Y$ is called a rectangular cozero set if it is expressed as $V=V_X\times V_Y$, where V_X and V_Y are cozero sets of X and Y respectively.

Theorem 2 (R. Pupier [3]). *If X is a locally compact Hausdorff space, then $\tau(X\times Y)=X\times\tau(Y)$ is valid for any space Y.*

The purpose of this paper is to show that the converse of Theorem 2 is valid in case X is a Tychonoff space. More generally, we can prove the following theorem.

Theorem 3. *Let X be a space. If $\tau(X)$ is not locally compact, then there exists a Hausdorff space Y such that $\tau(X\times Y)\neq\tau(X)\times\tau(Y)$.*

Combining Theorem 3 with Theorem 2, we have the following theorem.

Theorem 4. *Let X be a Tychonoff space. Then the following conditions are equivalent.*

(1) *X is locally compact.*

(2) *$\tau(X\times Y)=X\times\tau(Y)$ for any space Y.*

2. Preliminaries. Hereafter the symbol N denotes the set of all