44. On Closed Subvarieties of Parabolic Type in Certain Quasi-Projective Spaces of Hyperbolic Type

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Introduction. Recently S. Iitaka has developed a theory of logarithmic forms for algebraic varieties from proper birational geometric viewpoint and as an application he classified varieties of the form $V=(\boldsymbol{P}^n-$a union of hyperplanes) by means of logarithmic Kodaira dimension $\bar{\kappa}$ [1]. The present note is based on these results. We study closed subvarieties Γ's of V with $\bar{\kappa}(\Gamma)=0$ for V with $\bar{\kappa}(V)=n$. Recall that $\Gamma\simeq \boldsymbol{G}_m^r$, where $\boldsymbol{G}_m^r$ denotes the r-dimensional algebraic torus. For our purpose, the maximal ones among V's are useful.

1. Maximality. Let $V^n=\boldsymbol{P}^n(\boldsymbol{C})-L_0\cup\cdots\cup L_q$ where L_j's are distinct hyperplanes in $\boldsymbol{P}^n(\boldsymbol{C})$. The conditions in terms of coordinates for V^n with $\bar{\kappa}(V^n)=n$ can be described as follows. We may assume L_j is defined by $X_j=0$, $0\leqq j\leqq n$. For the other equations, putting $s=q-n$, define $I_1,\cdots,I_s\subset\{0,1,\cdots,n\}$ by $I_j=\{i\,|\,\text{coef. of } X_i \text{ of } L_{n+j} \text{ is not zero.}\}$ Then renumbering j if necessary, the following conditions 0) and 1) are satisfied.

0) $I_1\cup\cdots\cup I_s=\{0,1,\cdots,n\}$

1) $I_1\cup\cdots\cup I_{j-1}$ is not disjoint to I_j for $2\leqq j\leqq n$.

Proposition 1. *Let $\boldsymbol{C}a_j$ be the one dimensional subspace of $\boldsymbol{A}^{n+1}$ corresponding dually to L_j, $0\leqq j\leqq q$. Let (A^r,A^s) denote a pair of proper subspaces of A^{n+1} with $A^r\cap A^s=\{0\}$. Then V^n satisfies the above conditions* 0) *and* 1), *if and only if the following* (C) *holds.*

(C) *$A^r\cup A^s$ dose not contain all of $\boldsymbol{C}a_j$'s for any (A^r,A^s).*

Proposition 2. *If V^n with $\bar{\kappa}(V^n)=n$ is maximal, we can impose on V^n the following additional conditions* 2) *and* 3):

2) *There are s numbers, $2\leqq i(1)<\cdots<i(s)=n$, such that*

$$I_1=\{i\,|\,0\leqq i\leqq i(1)\}$$
$$I_j-I_1\cup\cdots\cup I_{j-1}=\{i\,|\,i(j-1)<i\leqq i(j)\},\ 2\leqq j\leqq s.$$

3) *Any two of I_j's never have only one common element.*

Proof of Proposition 2. 2) is obvious. Assume that $I_{j1}\cap I_{j2}=\{k\}$. Let $\boldsymbol{C}e_0,\cdots,\boldsymbol{C}e_n,\boldsymbol{C}a_1,\cdots,\boldsymbol{C}a_s$ be corresponding dually to $L_0,\cdots,L_n,L_{n+1},\cdots,L_{n+s}$. Let A_0 be the subspace of A^{n+1} spanned by $\{e_i\,|\,i\in I_{j1}\cup I_{j2}\}$. Since we are assuming that V^n is maximal, there is, by Proposition 1, (A^r,A^s) such that $\{e_0,\cdots,\check{e}_k,\cdots,e_n,a_1,\cdots,a_s\}\subset A^r\cup A^s$. This also induces a splitting $(A_0\cap A^r,A_0\cap A^s)$ for $\{e_i\,|\,i\in I_{j1}\cup I_{j2},\,i\neq k\}\cup\{a_{j1},a_{j2}\}$ in