42. *On the Meromorphy of Euler Products*

By Nobushige KUROKAWA
Department of Mathematics, Tokyo Institute of Technology

(Communicated by Kunihiko KODAIRA, M. J. A., June 15, 1978)

Introduction. We extend ordinary L-functions slightly and study their meromorphy. For simplicity we describe here the results on Euler products of Artin type which are contained in Part I of [3]. In Parts II and III of [3] we have some generalizations and modifications. Detailed proofs are described in [3].

§ 1. Euler products of Artin type. Let F be a finite extension of the rational number field $\boldsymbol{Q}$, K/F a finite Galois extension with the Galois group $G=\mathrm{Gal}\,(K/F)$, $R(G)$ the character ring of G (i.e. the ring of virtual characters of G; representations are over the complex number field $\boldsymbol{C}$). For $g \in G$ (or for the conjugacy class of G containing g) and for $H(T) \in 1+T\cdot R(G)[T]$ where T is an indeterminate, we denote by $H_g(T) \in 1+T\cdot \boldsymbol{C}[T]$ the polynomial obtained from $H(T)$ by taking the values of the coefficients at g. For each prime ideal $\mathfrak{p}$ of F unramified in K/F, let $\alpha(\mathfrak{p})$ denote the Frobenius conjugacy class $\left[\dfrac{K/F}{\mathfrak{P}}\right]$ in G, where $\mathfrak{P}$ is a prime ideal of K dividing $\mathfrak{p}$. We define $L(s,H) = \prod_{\mathfrak{p}} H_{\alpha(\mathfrak{p})}(N(\mathfrak{p})^{-s})^{-1}$ where $\mathfrak{p}$ runs over all prime ideals of F unramified in K/F.

We say $H(T) \in 1+T\cdot\boldsymbol{C}[T]$ is unitary if there exists a (complex) unitary matrix M such that $H(T)=\det(1-MT)$. We say $H(T)=1$ is unitary. For an Euler product over F ($F/\boldsymbol{Q}$ being a finite extension) $L(s,\boldsymbol{H})=\prod_{\mathfrak{p}} H_{\mathfrak{p}}(N(\mathfrak{p})^{-s})^{-1}$ with $\boldsymbol{H}=(H_{\mathfrak{p}})_{\mathfrak{p}}$, $H_{\mathfrak{p}}(T) \in 1+T\cdot\boldsymbol{C}[T]$, where $\mathfrak{p}$ runs over all prime ideals of F, we say $L(s,\boldsymbol{H})$ is unitary if $H_{\mathfrak{p}}(T)$ are unitary for all $\mathfrak{p}$. In general if $H_{\mathfrak{p}}(T)$ is not defined for a prime ideal $\mathfrak{p}$ of F, then we consider $H_{\mathfrak{p}}(T)=1$. We remark that the unitariness of $L(s,\boldsymbol{H})$ is not altered when we consider $L(s,\boldsymbol{H})$ as an Euler product over $\boldsymbol{Q}$ in the natural way. More precisely if F_0 is a subfield of F, then we can consider $L(s,\boldsymbol{H})$ as an Euler product over F_0 in the natural way as follows: for each prime ideal $\mathfrak{q}$ of F_0, put $H_{\mathfrak{q}}(T)=\prod_{\mathfrak{p}|\mathfrak{q}} H_{\mathfrak{p}}(T^{f(\mathfrak{p}|\mathfrak{q})})$ where $\mathfrak{p}$ runs over all prime ideals of F dividing $\mathfrak{q}$ and $f(\mathfrak{p}|\mathfrak{q})$ is the relative degree of $\mathfrak{p}$ over $\mathfrak{q}$, then $L(s,\boldsymbol{H})=L(s,\boldsymbol{H}_0)$ with $\boldsymbol{H}_0=(H_{\mathfrak{q}})_{\mathfrak{q}}$. Under this process the unitariness is not altered. It may be remarked that the unitariness is an analogue of the (normalized) "Riemann-Ramanujan-Weil conjecture" or "temperedness" for some arithmetic objects.