24. Hölder Conditions for the Local Times of Certain Gaussian Processes with Stationary Increments

By Norio KÔNO
Institute of Mathematics, Yoshida College, Kyoto University

(Communicated by Kôsaku YOSIDA, M. J. A., June 14, 1977)

1. Let $\{X(t,\omega);\ -\infty<t<\infty\}$ be a path continuous centered stationary Gaussian process with the spectral density function $f(\lambda)$ given by

$$f(\lambda)=a^{2\alpha}\frac{\Gamma(\alpha+1/2)}{\Gamma(1/2)\Gamma(\alpha)}(\lambda^2+a^2)^{-(\alpha+1/2)},\qquad 0<\alpha<1/2.$$

Then owing to Berman's result [2], there exists the local time $\psi(x,t,\omega)$ of $X(t,\omega)$ which is jointly continuous in x and t almost surely. For the local Hölder conditions of this local time, Davies [3] has proved the following:

$$0<c_1\leq\overline{\lim_{h\downarrow 0}}\frac{|\psi(X(t),t+h,\omega)-\psi(X(t),t,\omega)|}{h^{1-\alpha}(\log\log 1/h)^{\alpha}}\leq c_2<+\infty$$

for almost all ω.

We will extend his result to more wide class of Gaussian processes with stationary increments. We will give not only a local Hölder condition but also a uniform Hölder condition with respect to the upper bound. As for the lower bound, it is still open problem for our class.

2. Let $\{X(t,\omega);\ 0\leq t\leq 1\}$ be a path continuous centered Gaussian process with stationary increments: $E(X(t)-X(s))^2=\sigma^2(|t-s|)$. We assume the following:

(1) $\sigma(x)$ is a non-decreasing continuous nearly regular varying function with index α, $0<\alpha<1$, i.e. there exist two positive constants c and c', and also a slowly varying function $s(x)$ such that

$$cx^{\alpha}s(x)\leq\sigma(x)\leq c'x^{\alpha}s(x),$$

(2) $x/\sigma(x)$ is non-decreasing,

(3) $\sigma(x)$ is differentiable for $x>0$ with the derivative $\sigma'(x)$ such that

$$\sigma'(x)\leq\beta\sigma(x)/x,\quad \beta<1,\quad x>0.$$

(4) Denote by A_{2n} the correlation matrix $(r_{i,j})_{i,j=1}^{2n}$:

$$r_{i,j}=E\left[\frac{(X(t_i)-X(t_{i-1}))(X(t_j)-X(t_{j-1}))}{\sigma(t_i-t_{i-1})\sigma(t_j-t_{j-1})}\right],\qquad i,j=1,\cdots,2n,$$

for a partition $0=t_0<t_1<\cdots<t_{2n}\leq 1$. Then there exist a positive constant c_3 and a positive integer n_0 such that

$$\det A_{2n}\geq c_3^{2n}$$

holds for any partition of $[0,1]$ and any $n\geq n_0$.