53. On Bounded Sets of Holomorphic Germs

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Let E, F be separated, complex locally convex spaces, $\mathcal{H}(U; F)$ and $\mathcal{H}(K; F)$ denote the spaces of holomorphic mappings on an open subset U of E and of holomorphic germs on a compact subset K of E, respectively, endowed with their natural topologies (see Barroso [1], Mujica [4], Nachbin [5]). It is interesting to characterize the bounded subsets of $\mathcal{H}(K; F)$ in terms of the successive differentials. Such a characterization would be useful, by example, in the study of sequential convergence in $\mathcal{H}(K; F)$. Let $\mathcal{F}$ be a subset of $\mathcal{H}(K; F)$, Γ a family of seminorms defining the topology of F. We say that $\mathcal{F}$ has an estimate for the differentials in K if there exist a continuous seminorm α on E, real numbers $C>0$, $c>0$ such that for every β in Γ we have:

$$\sup_{x\in K}\left\|\frac{1}{m!}\hat{d}^m f(x)\right\|_{\alpha\beta}\leq Cc^m \qquad \text{for every } \tilde{f}\in\mathcal{F},\ f\in\tilde{f},\ m\in N.$$

One knows that an estimate for the differentials in K is not a sufficient condition for boundedness in $\mathcal{H}(K; F)$, but a bounded subset of $\mathcal{H}(K; F)$ has an estimate for the differentials when E and F are Banach spaces (Chae [2] and Wanderley [6]). Zame [7] showed that, under a weak local connectedness assumption on K, when K is a compact subset of C^n, an estimate for the differentials in K implies boundedness in the space $\mathcal{H}(K)$. The arguments used by Zame can be used in the general case.

Definition 1. Let X be a topological space, K a compact subset of X. We consider the following equivalence relation on X: $x, y\in X$, $x\sim y$ iff $x, y\in K$ or $x=y$. We denote by X/K the quotient space endowed with its natural topology, and K/K the equivalence class of an element of K.

Definition 2. Let X and K be as above. We say that K is of type LQC if, for every $x\in K$, there exists a sequence $K_1\subset\cdots\subset K_n$ of compact connected subsets of K such that $K_1=\{x\}$, K_{i+1}/K_i is locally connected for $i=1,\cdots,n-1$ and K/K_n is locally connected in K_n/K_n. We say that K is of type QC if there exists a sequence $K_1\subset\cdots\subset K_n$ of compact connected subsets of K such that K_1, K_{i+1}/K_i (for $i=1,\cdots,n-1$), K/K_n are locally connected. If K is locally connected then K is a compact of types LQC and QC. The class of compact subsets of E which