50. Nonlinear Parabolic Variational Inequalities with Time-dependent Constraints

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Let H be a (real) Hilbert space with the inner product $(\cdot,\cdot)_H$ and norm $\|\cdot\|_H$ in H, and X a uniformly convex Banach space with the strictly convex dual X^*, natural pairing $(\cdot,\cdot)_X: X^*\times X\to R^1$ and with norm $\|\cdot\|_X$ in X. Suppose that X is a dense subspace of H and the natural injection from X into H is continuous. Then, identifying H with its dual in terms of the inner product $(\cdot,\cdot)_H$, we have the relation $X\subset H\subset X^*$ where H is dense in X^*. Let $0<T<\infty$ and $2\leqq p<\infty$ with $1/p+1/p'=1$, and put $\mathcal{H}=L^2(0,T;H)$ and $\mathcal{X}=L^p(0,T;X)$ with $\mathcal{X}^*=L^{p'}(0,T;X^*)$; the natural pairing between $\mathcal{X}^*$ and $\mathcal{X}$ is denoted by $(\cdot,\cdot)_{\mathcal{X}}$ as well.

We are given a family $\{K(t);0\leqq t\leqq T\}$ of closed convex subsets of X satisfying that

(KI) *for each* $r\geqq 0$ *there are real-valued functions* $\alpha_r\in W^{1,2}(0,T)$ *and* $\beta_r\in W^{1,1}(0,T)$ *with the following property: for each* $s,t\in[0,T]$ *with* $s\leqq t$ *and* $z\in K(s)$ *with* $\|z\|_H\leqq r$ *there exists* $z_1\in K(t)$ *such that*

$$\|z_1-z\|_H\leqq|\alpha_r(t)-\alpha_r(s)|(1+\|z\|_X^{p/2})$$

and

$$\|z_1\|_X^p-\|z\|_X^p\leqq|\beta_r(t)-\beta_r(s)|(1+\|z\|_X^p).$$

We put $K_H=$the closure of $K(0)$ in H and $\mathcal{K}=\{v\in\mathcal{X};v(t)\in K(t)$ for a.e. $t\in[0,T]\}$.

We are also given a family $\{A(t);0\leqq t\leqq T\}$ of (nonlinear) operators from $D(A(t))=X$ into X^* such that

(AI) $\mathcal{A}$ *defined by* $[\mathcal{A}v](t)=A(t)v(t)$ *is an operator from* $D(\mathcal{A})=\mathcal{X}$ *into* $\mathcal{X}^*$ *and maps bounded subsets of* $\mathcal{X}$ *into bounded subsets of* $\mathcal{X}^*$;

(AII) *for each* $h\in\mathcal{X}$ *there are a positive number* c_0 *and a function* $c_1\in L^1(0,T)$ *satisfying*

$$(A(t)z,z-h(t))_X\geqq c_0[z]_X^p-c_1(t)\qquad a.e.\ on\ [0,T]$$

for all $z\in X$, *where* $[\cdot]_X$ *is a seminorm on* X *such that* $[\cdot]_X+\|\cdot\|_H$ *gives a norm on* X *equivalent to* $\|\cdot\|_X$.

With the above notation, given $f\in\mathcal{X}^*$ and $u_0\in K_H$, our problem $(V_s;f,u_0)$ is to find a function $u\in\mathcal{K}$ such that

(i) $u'\,(=du/dt)\in\mathcal{X}^*$ and $(u'+\mathcal{A}u-f,u-v)_{\mathcal{X}}\leqq 0$ for all $v\in\mathcal{K}$;

(ii) $u(0)=u_0$ (note that $u\in C([0,T];H)$ if $u\in\mathcal{K}$ and $u'\in\mathcal{X}^*$).