# 42. *Studies on Holonomic Quantum Fields. III*

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In this note we report along with [1] the work presented in [2]. Further results along the present line will be given in subsequent papers.

We follow the same notations as in [1] and [3] unless otherwise stated. In this article, along with the 2-dimensional space-time ($=$Minkowski 2-space) and its complexification, to be denoted by $X^{\mathrm{Min}}$ and $X^{C}$ respectively, we also deal with the Euclidean 2-space $X^{\mathrm{Euc}}$ consisting of complex Minkowski 2-vectors $x\in X^{C}$ such that $x^0\ (=-ix^2)\in i\boldsymbol{R}$ and $x^1\in\boldsymbol{R}$, i.e. such that $\mp x^{\mp}\ (=(\mp x^0+x^1)/2)$ are complex conjugate to each other; we have $z=-x^-$, $\bar z=x^+$, $\partial_z=\partial/\partial z$ and $\partial_{\bar z}=\partial/\partial\bar z$.

**1.** Let $W$ be an orthogonal vector space, and $W=V^\dagger\oplus V$ be its decomposition into two holonomic subspaces with basis $(\psi^\dagger_\mu)$ and $(\psi_\mu)$ as in §2 [3]. $V$ (resp. $V^\dagger$) generates maximal left (resp. right) ideal $A(W)V$ (resp. $V^\dagger A(W)$) of the Clifford algebra $A(W)$. The quotient modules $A(W)/A(W)V$ and $A(W)/V^\dagger A(W)$ are generated by the residue class of 1 modulo $A(W)V$ resp. $V^\dagger A(W)$ (which we shall denote by $|\mathrm{vac}\rangle$ and $\langle\mathrm{vac}|$ respectively after physicists' notation) and coincide with $A(V^\dagger)|\mathrm{vac}\rangle$ and $\langle\mathrm{vac}|A(V)$ since we have $V|\mathrm{vac}\rangle=0$ and $\langle\mathrm{vac}|V^\dagger=0$. Otherwise stated, they are respectively spanned by elements of the form $|\nu_n,\cdots,\nu_1\rangle\underset{\mathrm{def}}{=}\psi^\dagger_{\nu_n}\cdots\psi^\dagger_{\nu_1}|\mathrm{vac}\rangle$ and $\langle\nu_1,\cdots,\nu_n|\underset{\mathrm{def}}{=}\langle\mathrm{vac}|\psi_{\nu_1}\cdots\psi_{\nu_n}$, $n=0,1,2,\cdots$, and indeed these elements constitute mutually dual basis of both spaces: $\langle\mu_1,\cdots,\mu_m|\nu_n,\cdots,\nu_1\rangle=0$ if $m\neq n$, $=\det(\delta_{\mu_i\nu_j})$ if $m=n$.

Let $g$ be an element of the Clifford group $G(W)$. The rotation in $W$ induced by $g$, $T_g: w\mapsto gwg^{-1}$, is even or odd (i.e. $\det T_g=+1$ or $-1$) according as $\mathrm{corank}\,T_4=$even or odd; in particular for a generic even/odd $g\in G(W)$ we have $\mathrm{corank}\,T_4=0/1$ and expression (3)/(4) in [3] for $N(g)$. An element $w\in W$ itself belongs to $G(W)$ if and only if $\langle w,w\rangle\neq0$, in which case we have $wg\in G(W)$. First consider an even generic $g$, so that we have, with the abbreviation $\langle g\rangle\underset{\mathrm{def}}{=}\langle\mathrm{vac}|g|\mathrm{vac}\rangle$,

$$N(g)=\langle g\rangle e^L,\qquad L=\frac{1}{2}(\psi^\dagger\psi)\begin{pmatrix}S_1-1 & S_2\\ S_3 & S_4-1\end{pmatrix}\begin{pmatrix}{}^t\psi\\ -{}^t\psi^\dagger\end{pmatrix}\tag{21}$$

$${}^tS_1=S_4,\quad {}^tS_2=-S_2,\quad {}^tS_3=-S_3$$

where $S_g=\begin{pmatrix}S_1 & S_2\\ S_3 & S_4\end{pmatrix}$ is related to $T_g=\begin{pmatrix}T_1 & T_2\\ T_3 & T_4\end{pmatrix}$ through the reciprocal formulas