38. *On Certain Spaces Admitting Concircular Transformations.*

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(Comm. by Z. SUETUNA, M. J. A., June 16, 1949.)

§0. *Introduction.* One of the present authors had studied the conformal transformations

$$\bar{g}_{\mu\nu}=\rho^2 g_{\mu\nu} \tag{0.1}$$

of Riemannian metrics which change any Riemannian geodesic circle

$$\frac{\delta^3 x^\lambda}{ds^3}+\frac{dx^\lambda}{ds}g_{\mu\nu}\frac{\delta^2 x^\mu}{ds^2}\frac{\delta^2 x^\nu}{ds^2}=0 \tag{0.2}$$

into a Riemannian geodesic circle, and called such transformations concircular transformations.[1]

In order that the conformal transformation (0.1) be a concircular one, it is necessary and sufficient that the function ρ satisfies the differential equations

$$\rho_{\mu\nu}\equiv\rho_{\mu;\nu}-\rho_\mu\rho_\nu+\frac{1}{2}g^{\beta\gamma}\rho_\beta\rho_\gamma g_{\mu\nu}=\phi g_{\mu\nu}, \tag{0.3}$$

where $\rho_\mu=(\log\rho)_{;\mu}$ and the semi-colon denotes the covariant differentiation with respect to the Christoffel symbols $\{^{\lambda}_{\mu\nu}\}$, ϕ being a certain scalar.

If we put

$$\sigma=\frac{1}{\rho}, \tag{0.4}$$

the condition (0.3) may also be written as

$$\sigma_{\mu;\nu}=\alpha g_{\mu\nu}; \tag{0.5}$$

where $\sigma_\mu=\sigma_{;\mu}$ and α is a certain scalar.

If the partial differential equations (0.3) or (0.5) admit a solution, the family of hypersurfaces defined by ρ=constant or σ=constant are totally umbilical and their orthogonal trajectories are geodesic Ricci curves.

Conversely, if a Riemannian space contains a family of ∞^1 totally umbilical hypersurfaces whose orthogonal trajectories are geodesic Ricci curves, the space admits a concircular transformation.

In the present note, we shall study the spaces which admit the concircular transformation and satisfy some additional conditions.

§1. We shall first consider a Riemannian space which admits a

(1) K. Yano : Concircular Geometry, I, II, III, IV, V. Proc. 16(1940), 195-200 ; 354-360 ; 442-445 ; 505-511 ; 18 (1942), 446-451.