20. Theory of Invariants in the Geometry of Paths. II. Equivalence Problems.

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§ 1. The method hitherto utilized in the case of the generalized rheonomic geometry of paths is, after some suitable modifications, available to geometries of paths such as ordinary, intrinsic, and rheonomic. This is due to the fact that the transformation group (I, 0.2) of the generalized rheonomic geometry is an extension, in some meaning, of the transformation group (I, i), (I, ii), (I, iii) on which the other geometries are based. However the results obtained by our method are somewhat complicated for these geometries compared with the results hitherto already known.

Concerning the intrinsic geometry of paths we give here some brief additional notes for the results of A. Kawaguchi—H. Hombu [5][1]. Under the intrinsic group (I, ii), as in the case of generalized rheonomic group G (I, 0.2), the Pfaffians $\mathfrak{d}x^{(r)i}$ $(r=0, 1, \dots, m-1)$ defined by (I, 2.2), instead of the ordinary differentials $dx^{(r)i}$ $(r=0, 1, \dots, m-1)$, are subject to the transformation law (I, 2.3). Hence we must use $\mathfrak{d}x^{(r)i}$ instead of $dx^{(r)i}$ in the fundamental theorem[2] concerning the covariant differential of the line-element. Owing to this modification the covariant derivative[3] ∇v^i of a vector field v^i of weight p turns to be our (I, 5.6). As to the results of S. Hokari [3] the same considerations are necessary.

§ 2. In this paragraph we study the equivalence problem in the geometry of paths in the case of generalized rheonomic geometry.

Let us consider two systems of paths defined by

$$(2.1)\qquad \begin{aligned} &\text{(i)}\quad x^{(m)i}+H^i(t, x, x^{(1)}, \dots, x^{(m-1)})=0\,,\\ &\text{(ii)}\quad \bar{x}^{(\overline{m})\alpha}+\overline{H}^{\alpha}(\bar{t}, \bar{x}, \bar{x}^{(\overline{1})}, \dots, \bar{x}^{(\overline{m-1})})=0\,. \end{aligned}$$

If there exists a transformation belonging to the generalized rheonomic group G such that the relations (I, 2.1) hold between the two sets of quantities

$$(t, x^i, x^{(1)i}, \dots, x^{(m-1)i}, H^i) \quad\text{and}\quad (\bar{t}, \bar{x}^{\alpha}, \bar{x}^{(\overline{1})\alpha}, \dots, \bar{x}^{(\overline{m-1})\alpha}, \overline{H}^{\alpha})\,,$$

we say that the two systems of paths are equivalent to each other

1) Numbers in brackets refer to the bibliography at the end of the paper.
2) [5], P. 58, Theorem 16.
3) [5], P. 60, (3.21).