155. Dirichlet Problem on Riemann Surfaces. I

(Correspondence of Boundaries)

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Let $\underline{R}$ be an open abstract Riemann surface and let $\{\underline{R}_n\}$ ($n=1, 2, \dots$) be an exhaustion with compact relative boundaries $\{\partial\underline{R}_n\}$.[1] Then $\underline{R}-\underline{R}_n$ is composed of a finite number of disjoint non compact subsurfaces $\{G^i_n\}$ ($i=1, 2, \dots, i_n$: $n=1, 2, \dots$). Let $\{G^i_n\}$ be a sequence of non compact surfaces such that $G^i_n\supset G^{i'}_{n+1}\dots$, $\bigcap_n G^i_n=0$. Two sequences $\{G^i_n\}$ and $\{G^{i'}_m\}$ are called equivalent, if and only if, for any given number m, there exists a number n such that $G^{i'}_m\supset G^i_n$ and vice versa. We correspond an ideal point (component) to a class of equivalent sequences and denote the set of all ideal boundary points by B. A topology is introduced on $\underline{R}+B$ by the completion of $\underline{R}$. It is clear that $\underline{R}+B$ is closed, compact and that B is totally disconnected. This topology restricted in $\underline{R}$ is homeomorphic to the original topology. We call this topology A-topology and denote $\underline{R}+B$ by $\underline{R}^*$.[2]

Let R be an abstract Riemann surface given as a covering surface over $\underline{R}$. We define the distance of two points p_1 and p_2 of R by $\inf(\delta(p_1, p_2))$, where $\delta(p_1, p_2)$ is the diameter of the projection of a curve on R connecting p_1 and p_2, and define the accessible boundary points of R by the completion of R with respect to this metric. When a continuous curve L on R converges to the boundary of R and the projection of L on $\underline{R}$ tends to a point of $\underline{R}^*$, we say that L determines an accessible boundary point (abbreviated to A.B.P.). It is well known that these two definitions are equivalent.

In this paper we suppose that $\underline{R}$ is a null-boundary Riemann surface.

Lemma 1.1. Let R be a covering surface over $\underline{R}$, let $\underline{z}=f(z)$ ($\underline{z}\in\underline{R}$, $z\in R$) be the mapping function from R into $\underline{R}$ and let L be a curve on R which determines an A.B.P. whose projection on B is $\underline{z}_0$. Suppose that R does not cover a subset of positive capacity of $\underline{R}$. We map the universal covering surface R^∞ conformally onto the unit circle U_ξ: $|\xi|<1$ by $\xi=\varphi(z)$. If the image l[3] *of L in U_ξ tends to a point ξ_0 on $|\xi|=1$, then the composed function $\underline{z}=f(\varphi^{-1}(\xi))$ has the*

1) Thought this paper, we denote a relative boundary of G by ∂G.

2) It is clear that a metric introduced in A-topology.

3) In this case, it is proved that l does not osciliate.