105. Relations among Topologies on Riemann Surfaces. IV

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(Comm. by K. KUNUGI, M.J.A., Oct. 12, 1962)

Example 4. Let $\mathfrak{R}$ be a circle $|z+1|<1$. Let R_n be a domain such that R_n: $\frac{1}{2^n}\geqq|z|\geqq\frac{1}{2^{n+1}}$, $|\arg z|\leqq\frac{\pi}{16}$ and put $\sum\limits_{n=1}^{\infty}R_n=R$ and $D=\mathfrak{R}-R$. Domain $\mathfrak{D}$. Let A_n and Γ_n be domains as follows:

$$A_n:\frac{1}{2^{n+1}}+a_n>|z|>\frac{1}{2^n}\ \text{ and }\ a_n<\frac{1}{3\times 2^{n+1}},\ |\arg z|<\frac{\pi}{16},$$

$$\Gamma_n:\frac{1}{2}\left(\frac{1}{2^n}+\frac{1}{2^{n+1}}\right)\geqq|z|\geqq\frac{1}{2}\left(\frac{1}{2^{n+1}}+\frac{1}{2^{n+2}}\right),\ |\arg z|\leqq\frac{\pi}{8},$$

where a_n will be determined. Then $\Gamma_n\supset A_n$ and dist $(\partial\Gamma_n, A_n)>0$. Let $G(z, p_0, \mathfrak{R})$ be the Green's function of $\mathfrak{R}$, where $p_0=-\frac{3}{2}$. Put $M_n=\max G(z, p_0, \mathfrak{R})$ on $\partial R_n+\partial R_{n+1}$. Let $w(z, A_n, D)$ be the harmonic measure of A_n-D relative to D. Now D is simply connected and dist $(\partial\Gamma_n, A_n)>0$. Hence by Lemma 3 or 5 we can find a constant a_n such that

$$M_n w(z, A_n, D)\leqq\frac{1}{4^n}G(z, p_0, D)\ \text{ on }\ \partial\Gamma_n. \tag{15}$$

We suppose a_n is defined as above. Put $\mathfrak{D}=\mathfrak{R}-R+\sum\limits_{n=1}^{\infty}A_n$. Now $M_n w(z, A_n, D)=0=\frac{1}{4^n}G(z, p_0, D)$ on $\partial D-\Gamma_n$. Hence by the maximum principle $M_n w(z, A_n, D)\leqq\frac{1}{4^n}G(z, p_0, D)$ in $D-\Gamma_n$. By $M_n\geqq G(z, p_0, \mathfrak{R})$ $\geqq G(z, p_0, \mathfrak{D})$ on ∂A_n we have $M_n\geqq M_n w(z, A_n, D)+G(z, p_0, D)\geqq G(z, p_0, \mathfrak{D})$

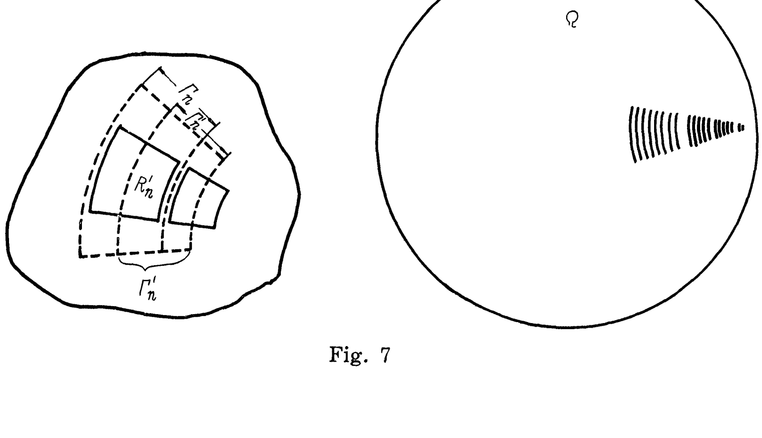

Fig. 7