138. *Closures and Neighborhoods in Certain Proximity Spaces*

By Eiichi HAYASHI
Nagoya Institute of Technology

(Comm. by Kinjirô KUNUGI, M.J.A., Sept. 12, 1967)

In the paper [1], we defined a binary relation δ, called a paraproximity, for a pair of subsets of a point set R. We there proved that a paraproximity yields a completely normal space ([1] Theorem 1). Further we showed that, for a pair of subsets A and B of a paraproximity space R, $(A, B)\in\delta$ implies $A\cap\bar{B}\neq\emptyset$ ([1] Theorem 2). In the present paper we show that the converse of Theorem 2 holds. Hence a paraproximity structure which is compatible with the topology is uniquely determined. The remaining parts of this paper are devoted to the study of the neighborhood.

First we restate the definition of a paraproximity. By a *paraproximity* on a set R we mean a binary relation δ for pairs of subsets of R satisfying the following axioms:

Axiom (1). For every $A\subset R$, $(A, \emptyset)\notin\delta$, and $(\emptyset, A)\notin\delta$. (We add the latter condition $(\emptyset, A)\notin\delta$ to Axiom (1) of [1].)

Axiom (2). $(A, B\cup C)\in\delta$ if and only if either $(A, B)\in\delta$ or $(A, C)\in\delta$.

Axiom (3). For an arbitrary index set Λ, $(\bigcup_{\lambda\in\Lambda}A_\lambda, B)\in\delta$ if and only if there exists an index $\mu\in\Lambda$ satisfying the relation $(A_\mu, B)\in\delta$.

Axiom (4). For arbitrary two points $a, b\in R$, $(\{a\}, \{b\})\in\delta$ if and only if $a=b$.

Axiom (5). If $(A, B)\notin\delta$ and $(B, A)\notin\delta$, then there exist two disjoint subsets U and V satisfying:

$$(A, R-U)\notin\delta, \qquad (U, R-U)\notin\delta:$$
$$(B, R-V)\notin\delta, \qquad (V, R-V)\notin\delta.$$

We note that the next lemma (Steiner [3]) follows from Axiom (3).

Lemma 1. $(A, B)\in\delta$ *if and only if* $(\{x\}, B)\in\delta$ *for some* x *in* A.

Lemma 2. *If* $(\{x\}, A)\notin\delta$ *then* $(A, \{x\})\notin\delta$.

Proof. *If* $(\{x\}, A)\notin\delta$, then $x\notin A$ by [1, Lemma 3]. Suppose that $(A, \{x\})\in\delta$. Then, by Lemma 1, there is a point a in A such that $(\{a\}, \{x\})\in\delta$. From Axiom (4) follows $a=x$ which is a contradiction.

1. Let R be a set with a paraproximity δ. A set $B\subset R$ is said to be a *paraproximal neighborhood* of a set $A\subset R$ (notation: