45. On Potential Kernels Satisfying the Complete Maximum Principle

By Ryōji KONDŌ
Department of Mathematics, Shizuoka University, Shizuoka

(Comm. by Zyoiti SUETUNA, M. J. A., April 12, 1968)

Let $(E, \mathcal{E})$ be a measurable space and V a proper kernel on $(E, \mathcal{E})$ which satisfies the complete maximum principle. It is known that if $V1$ is bounded, there then exists a sub-Markov resolvent $(V_p)_{p>0}$ such that

$$V = \lim_{p\to 0} V_p \tag{1}$$

(see [4, p. 206]). On the other hand, if $V1$ is unbounded, there is such a kernel V for which the condition (1) is never satisfied by any sub-Markov resolvent $(V_p)_{p>0}$ (for an example, see also [4, p. 206]).

In this note we shall give a *sufficient* condition under which the kernel V can be expressed in the form (1) by a sub-Markov resolvent $(V_p)_{p>0}$. The condition is stated in terms of the pseudo-réduite and it is similar to that of Theorem 7 of Meyer [5].*) Our result contains Theorem II of Lion [3] as a special case.

1. **Preliminary results.** Throughout this note notations and terminology are taken from Meyer [4]. We will omit the definitions of a *proper* [resp. *sub-Markov*] kernel, a *sub-Markov resolvent* (we shall call it simply a *resolvent*) and a *supermedian function* with respect to a resolvent. A subset of E and a function on E are always assumed to be $\mathcal{E}$-measurable, so we will omit the phrase "$\mathcal{E}$-measurable".

Let A be a subset of E and h a supermedian function with respect to a resolvent $(V_p)_{p>0}$. Then the collection of supermedian functions that dominate h on A has the smallest element, which will be called the *pseudo-réduite* of h on A and denoted by $H_A h$ [4, p. 200]. A resolvent $(V_p)_{p>0}$ is said to be *closed* if the kernel V_0 defined by $V_0 = \lim_{p\to 0} V_p$ is proper. If $(V_p)_{p>0}$ is closed and $V_0 f$ ($f \geqq 0$) is finite, then the function $V_0 f$ is supermedian with respect to $(V_p)_{p>0}$. If the support of f is contained in A, then $H_A V_0 f = V_0 f$ [5, p. 231].

Let U be any proper kernel on $(E, \mathcal{E})$. A non-negative function

*) Meyer discussed the following problem and gave a necessary and sufficient condition for the kernel U. "When is the proper kernel U generated by a sub-Markov kernel P in the sense $U = \sum_{n=0}^{\infty} P^n$". This is closely connected to our problem.