# 43. On Closed Mappings of Generalized Metric Spaces

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N. S. Lašnev [5] proved the following theorem:

*Let $f$ be a closed continuous map ($=$mapping) from a metric space $X$ onto a space $Y$. Then $Y=\bigcup_{n=0}^{\infty} Y_n$, where $Y_n$ is a discrete subset of $Y$ for each $n>1$, and $f^{-1}(y)$ is compact for each $y \in Y_0$.*

This theorem was extended by several mathematicians, especially by A. Okuyama [7] to normal $\sigma$-space, by R. A. Stoltenberg [9] to normal semi-stratifiable space and by V. V. Filippov [1] to paracompact $M$-space. It is almost a surprising fact that under such general circumstances so many points of $Y$ have compact inverse images. Besides, this type of theorem is not a mere object of curiosity as shown by F. G. Slaughter [8] who used it to prove an interesting metrization theorem. Since, as well known, a regular space is metrizable if and only if it is $\sigma$ and $M$ and since every $\sigma$-space is semi-stratifiable, the above theorems by Okuyama and Filippov are extensions of Lašnev's theorem to two different directions while Stoltenberg's theorem generalizes Okuyama's. Thus it is natural to try to unify the two theorems of Stoltenberg and of Filippov. Although this attempt is not fully successful yet, we have made a partial success. In fact the purpose of this paper is to extend Lašnev's theorem to two classes of generalized metric spaces which contain all $M$-spaces as well as all semi-metric spaces. (Note that semi-metric$=$semi-stratifiable plus 1-st countable as proved by R. W. Heath [2].)

All spaces in the following discussions are $T_1$ except in Definitions, and all maps are continuous. $N$ denotes the sequence of natural numbers $\{1, 2, 3, \cdots\}$, and a subsequence means an infinite subsequence. As for general terminologies and symbols, see J. Nagata [6] and also the above mentioned references for more specialized terminologies.

Let us recall that a collection $\mathcal{U}$ of (not necessarily open) subsets of a space $X$ is called a *nbd base of a subset* $F$ of $X$ if the interior of each member of $\mathcal{U}$ contains $F$ and if for every open set $V$ containing $F$ there is a member of $\mathcal{U}$ which is contained in $V$ and also that a collection $\{F_\alpha \mid \alpha \in A\}$ of subsets of $X$ is called to be *cushioned* in a collection $\{U_\alpha \mid \alpha \in A\}$ of subsets if $(\cup\{F_\alpha \mid \alpha \in A'\})^- \subset \cup\{U_\alpha \mid \alpha \in A'\}$ for every subset $A'$ of $A$. One of our new classes of generalized metric spaces is defined as follows.