36. *On Kodaira Dimensions of Certain Algebraic Varieties*

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1. In this note we give our results on *Kodaira dimensions* of subvarieties of abelian varieties and of varieties which are images of rational maps of abelian varieties. Details will be published elsewhere.

2. In this note all algebraic varieties are assumed to be irreducible and defined over $\boldsymbol{C}$.

Let V be a complete algebraic variety of dimension n, and let $\tilde{V}$ be a non-singular model of V. For any positive integer m, the *m-genus* $p_m(V)$ and the *irregularity* $q(V)$ of V are defined by

$$p_m(X)=\dim_{C} H^0(\tilde{V}, (\Omega^n_{\tilde{V}})^{\otimes m}),$$
$$q(V)=\dim_{C} H^0(V, \Omega^1_{\tilde{V}}),$$

where $\Omega^k_{\tilde{V}}$ is the sheaf of germs of holomorphic k forms on $\tilde{V}$. Instead of $p_1(V)$, we use the notation $p_g(V)$ and call it the *geometric genus* of V. $p_m(V)$ and $q(V)$ are independent of the choice of the non-singular model $\tilde{V}$ of V.

S. Iitaka [1] proved that if there exists a positive integer m_0 such that the inequality $p_{m_0}(V)>1$ holds, then the inequality

$$\alpha m^{\kappa(V)} \leqq p_{m m_0}(V) \leqq \beta m^{\kappa(V)}$$

holds for every sufficiently large integer m, where α, β are positive numbers and $\kappa(V)$ is a positive integer uniquely determined by the plurigenera of V. When the inequality $p_m(V)\leqq 1$ holds for every positive integer m and the equality holds for at least one integer m, we define $\kappa(V)=0$. When all plurigenera vanish, we define $\kappa(V)=-\infty$.

As every plurigenus is a birational invariant of $V, \kappa(V)$ is also a birational invariant of V. $\kappa(V)$ is called the *Kodaira dimension* of the variety V.

3. Let B be a subvariety of an abelian variety A. Then we have $\kappa(B)\geqq 0$. This implies the well known fact that unirational and ruled varieties cannot be embedded into abelian varieties. For this subvariety B, we have the following theorem.

Theorem 1. *The following conditions are equivalent:*

1) $p_g(B)=1$

2) $\kappa(B)=0$

3) *The subvariety B is a translations of an abelian subvariety B' of A by an element $a \in A$.*

Corollary 1. *We assume that B goes through the origin of A.*