136. *Remarks on the Cesàro Summability of Divergent Series.*

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The object of this paper is to prove a converse of Cauchy's theorem concerning limit and give alternate proofs of Doetsch's theorem[1] and the well-known Cesàro-Tauberian theorem due to Hardy and Landau.

1. *Theorem* I. *If*

$$na_n \geqq (n-1)a_{n-1}, \qquad n>1, \tag{1}$$

then

$$\lim_{n\to\infty} \frac{a_1+a_2+\cdots\cdots+a_n}{n}=L \tag{2}$$

implies $\lim\limits_{n\to\infty} a_n=L$.

Proof. Since the sequence (na_n) is monotone increasing, its limit exists. If the limit of (na_n) is finite, then $a_n\to 0$, consequently L must be 0. In this case the theorem is evident. If $L\neq 0$, the limit of (na_n) can not be finite. Thus we have to discuss the case, where na_n tends to infinity.

Plainly we can suppose that a_n is positive for all n. For any positive number ε, there is an integer n_0 such that

$$\left|\frac{a_1+a_2+\cdots\cdots+a_n}{n}-L\right|<\varepsilon, \tag{3}$$

for $n\geqq n_0$. Let p be a fixed positive integer, then

$$\frac{a_1+a_2+\cdots\cdots+a_n+a_{n+1}+\cdots\cdots+a_{n+\left[\frac{n}{p}\right]}}{n+\left[\dfrac{n}{p}\right]}<L+\varepsilon,$$

for $n\geqq n_0$, where $[x]$ denotes the integral part of x.

From (3), we have

1) Doetsch: Über die Cesàrosche Summabilität bei Reihen und eine Erweiterung des Grenzwertbegriffs bei integrablen Funktionen. Math. Zeit. **11** (1921). See Nikola Obreschkoff: Über einige Sätz für Summierung von divergenten Reihen. Tôhoku Math. Journ. **32** (1930).