PAPERS COMMUNICATED

56. Note on Banach Spaces (III): A Proof of Tietze-Matsumura's Theorem.

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A subset S of a linear metric space E is called *locally convex*, if and only if, for every $p \in S$ there exists a sphere K with center p such that $S \frown K$ is convex. In the case of the Euclidean n-spaces H. Tietze[1] and S. Matsumura[2] proved, that every closed, connected and locally convex set is convex.

In the present note we extend this theorem into the following form:

Theorem. If E is uniformly convex and S is compact, closed and connected set, then the local convexity of S implies the convexity in the large.

To prove the theorem we choose a finite covering by spheres $\{K_i\}$[3] such that $S \frown K_i$ is convex; this is possible always, since the set S is compact. If a set $S \frown K_i \frown K_j$ for $i \neq j$ is non-void, then we call it a *shoal*. It is evident that a shoal is compact and closed.

On the other hand, we define a *bridge* as a continuous image of $[0, 1]$ to S, which contains only a finite number of line-segments — called *girders* —, pass through a shoal once only and joint points of girders — called *piles* — lie in shoals. For the sake of simplicity, we assume a shoal contains only one pile and even if a girder pass through a shoal, we join a pile on it.

Then obviously a bridge can be represented by an ordered set of piles and end-points such that

$$I = (p_0, p_1, \cdots, p_n).$$

Next, we define the *length of bridge* by

$$|I| = \sum_{i=1}^{n} |p_i - p_{i-1}|.$$

Since, as remarked above, all shoals are compact, we can find a bridge from a to b with minimal length. Hence to prove the theorem it is sufficient to show the following

Lemma. Every bridge with minimal length between two points of S is itself a line-segment.

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1) H. Tietze, Math. Zeits., **28** (1928), 697–707.
2) S. Matsumura (Nakajima), Tôhoku M. J., **28** (1928), 266–268.
3) We assume here K_i's sre closed spheres.