14. *Interpolation of Linear Operators in Lebesgue Spaces with Mixed Norm*

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The aim of this paper is to show that a bounded linear operator in the Lebesgue spaces $L^t(M^n; L^s(M^m))$ with mixed norm is bounded in the space $L^u(M^{m+n})$ under a condition on (s, t), where $1/u=(m/s+n/t)/(m+n)$. As applications we shall have a result on Riesz-Bochner summing operator and on the restriction problem of Fourier transform.

1. Notations. Let $(M, \mathcal{M}, \mu)$ and $(N, \mathcal{N}, \nu)$ be σ-finite measure spaces, and $(M_j, \mathcal{M}_j, \mu_j)$ $(j=0, 1, \cdots)$ be copies of $(M, \mathcal{M}, \mu)$. Let $d\geq 2$ and $(\bar{M}, \bar{\mathcal{M}}, \bar{\mu})$ be the product measure space $\prod_{j=0}^{d-1}(M_j, \mathcal{M}_j, \mu_j)$. For a subset $p=\{p_0, p_1, \cdots, p_{m-1}\}\subset\{0, 1, \cdots, d-1\}$ put

$$(M(p), \mathcal{M}(p), \mu(p))=\prod_{j\in p}(M_j, \mathcal{M}_j, \mu_j).$$

Thus

$$d\mu(p)(x_{p_0}, \cdots, x_{p_{m-1}})=d\mu_{p_0}(x_{p_0})\cdots d\mu_{p_{m-1}}(x_{p_{m-1}}) \quad\text{and}\quad d\bar{\mu}=d\mu(p)\times d\mu(p'),$$

where p' denotes the complement $\{0, 1, \cdots, d-1\}\backslash p$. $(\bar{N}, \bar{\mathcal{N}}, \bar{\nu})$ and $(N(p), \mathcal{N}(p), \nu(p))$ will be defined similarly.

Let $1\leq s, t<\infty$. $L^s(\bar{M})$ denotes the Lebesgue space with norm $\|f\|_s=\left(\int_{\bar{M}}|f|^s d\bar{\mu}\right)^{1/s}$ and $L^t(L^s)=L^t(M(p'); L^s(M(p)))$ the Lebesgue space with mixed norm

$$\|f\|_{(t,s;p)}=\left[\int_{M(p')}\left(\int_{M(p)}|f|^s d\mu(p)\right)^{t/s}d\mu(p')\right]^{1/t}.$$

The definition for the cases $s=\infty$ and/or $t=\infty$ will be modified obviously.

Let m and n be positive integers such that $d=m+n$. We define $u\geq 1$ by

$$1/u=(m/s+n/t)/d.$$

For $1\leq s\leq\infty$, s' will denote the conjugate exponent $s/(s-1)$.

P denotes the family $\{p\in\{0, 1, \cdots, d-1\}; \operatorname{card}(p)=m\}$ if $m\geq n$ and $P=\{0, 1, \cdots, d-1\}$ otherwise. Let $\{I_p; p\in P\}$ be a family of disjoint arcs in the unit circle of length $2\pi/\operatorname{card}(P)$.

2. Theorems.

Lemma 1. *Assume* $1\leq s\leq t\leq\infty$. *Let* w *and* f *be simple functions in* $(\bar{M}, \bar{\mathcal{M}}, \bar{\mu})$. *Then there exist functions* $W^z(x)$ *and* $F^z(x)$ *on* $\bar{M}$ *such that*

(i) $W^z(x)$ *and* $F^z(x)$ *are bounded and holomorphic in* $|z|<1$ *for every* $x\in\bar{M}$, *and measurable in* x *for every* $|z|<1$,

(ii) $W^0(x)=w(x)$ *and* $F^0(x)=f(x)$,

(iii) $\|W^z\|_{(t,s;p)}\leq\|w\|_u$ *for* $z\in\operatorname{int}(I_p)$ *and* $p\in P$,

and