102. On the Normality of Certain Product Spaces

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Let X be the image of a metric space R under a closed continuous mapping f and let Y be the image of a metric space S under a closed continuous mapping g. We shall be concerned with the normality of the product space $X\times Y$.

As is well known, the spaces X and Y are both paracompact and perfectly normal. But the topological product of two normal spaces is not normal in general. In fact, as the example given by E. Michael [2] shows, the product space $W\times Z$ is not necessarily normal, even if W is a hereditarily paracompact Hausdorff space with Lindelöf property and Z is a separable metric space.

K. Morita has given in [4] two closed continuous mappings whose product is not a closed mapping. It should be noted that one of these mappings is a perfect mapping, and hence the product of a closed continuous mapping and a perfect mapping is not always a closed mapping. Thus the normality of the product space $X\times Y$ is not concluded directly from the normality of the product space $R\times S$.

In this note, we shall establish the following:

Theorem 1. *If the space R is a locally compact metric space, then the product space $X\times Y$ is normal.*

1. Our proof will be based on the following theorems established by K. Morita in [5] and [3].

Theorem 2. *Let X be a paracompact normal space which is a countable union of locally compact closed subsets, and let Y be a paracompact normal space. Then the product space $X\times Y$ is paracompact and normal.*

Theorem 3. *Let X be a paracompact and perfectly normal space, which is a countable union of locally compact closed subsets and is also a countable union of closed metrizable subspaces. Let Y be a paracompact and perfectly normal space. Then the product space $X\times Y$ is paracompact and perfectly normal.*

Theorem 4. *Let f be a closed continuous mapping of a paracompact and locally compact Hausdorff space R onto another topological space X. Denote by X' be the set of all points x of X such that $f^{-1}(x)$ is not compact, and by X'' the set of all points x of X such that $\mathfrak{B}f^{-1}(x)$ is not compact. Then we have:*

(a) $X''\subset X'$;