

Split Semi-Biplanes in Antiregular Generalized Quadrangles

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Abstract

There are a number of important substructures associated with sets of points of antiregular quadrangles. Inspired by a construction of P. Wild, we associate with any four distinct collinear points p, q, r and s of an antiregular quadrangle an incidence structure which is the union of the two biaffine planes associated with $\{p, r\}$ and $\{q, s\}$. We investigate when this incidence structure is a semi-biplane.

1 Introduction

The definitions of both semi-biplanes and antiregular generalized quadrangles involve 0-2 conditions. That this is more than just a coincidence is shown in [4], where we investigate a first construction of semi-biplanes from anti-regular quadrangles.

It turns out that associated with any pair of non-collinear points in an antiregular generalized quadrangle is an incidence structure which is the union of semi-biplanes. All these semi-biplanes look very much like homology semi-biplanes, that is, projective planes that have been folded up using a homology involution. In [6] we prove that in the classical case and the topological case the resulting semi-biplanes are all homology semi-biplanes.

The definition of the semi-biplane associated with two non-collinear points still makes sense if we use a pair of collinear points instead. The resulting incidence structure is a biaffine plane, that is, a projective plane minus an incident point-line pair.

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