All 2-(21,7,3) designs are residual

Edward Spence

Abstract

In a previous classification of symmetric 2-$(31, 10, 3)$ designs it was discovered that the 151 pairwise non-isomorphic designs found yielded a total of 3809 residual 2-$(21, 7, 3)$ designs that were pairwise non-isomorphic. Here we report on a computer search for all 2-$(21, 7, 3)$ designs which showed that the 3809 obtained above constitute the complete set.

1 Introduction

By a 2-(v, k, λ) design we mean a pair $\mathcal{D} = (\mathcal{X}, \mathcal{B})$, where $\mathcal{X}$ is a set of v 'points' and $\mathcal{B}$ is a collection of b 'blocks' together with an incidence relation that satisfies the following conditions: each block is incident with k points and each pair of distinct points is incident with λ blocks. For more details and basic facts concerning these 2-(v, k, λ) designs see [1] and [5]. From a given symmetric ($b = v$) 2-(v, k, λ) design $\mathcal{D} = (\mathcal{X}, \mathcal{B})$ there is a way of constructing its *residual* design. This is obtained by fixing a block $B \in \mathcal{B}$ and taking $\mathcal{D}' = (\mathcal{X} \setminus B, \mathcal{B}')$, where $\mathcal{B}' = \{B' \setminus B : B' \in \mathcal{B}, B' \neq B\}$, and the incidence relation is that induced from $\mathcal{D}$. The parameters of the residual design are $(v - k, k - \lambda, \lambda)$. Any design with the parameters of a residual design is called *quasi-residual*. It is well-known [5, Theorem 16.1.3] that any quasi-residual design with $\lambda = 1$ or 2 is in fact residual, but when $\lambda > 2$ the situation is somewhat different. There is a 2-$(16, 6, 3)$ design, whose construction is due to Bhattacharya [2], and which is not the residual of a 2-$(25, 9, 3)$ design since it has two blocks that intersect in four points. In the Tables of [7] the three 'smallest' sets of parameters of 2-designs with $\lambda = 3$ that are quasi-residual designs are 2-$(8, 4, 3)$ (number 15), 2-$(16, 6, 3)$ (number 35)

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