High Singer invariant and equality of curvature

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Abstract

The author proves, by giving explicit examples, that the Singer invariant of a locally homogeneous Riemannian manifold can become arbitrarily high. In a second step it is shown that for each $k \in \mathbb{N}$ there exist pairs of nonisometric homogeneous Riemannian manifolds of Singer invariant k which have the same curvature up to order k.

1 Introduction

Let (M, g) be a locally homogeneous Riemannian manifold and let $V = T_pM$ be the tangent space at a point p which, by local homogeneity, can be chosen arbitrarily. Furthermore, let $\mathfrak{so}(V)$ be the Lie algebra of skew-symmetric endomorphisms of (V, g_p). For $k \in \mathbb{N}_0$ we define Lie subalgebras $\mathfrak{g}(k)$ of $\mathfrak{so}(V)$ by

$$\mathfrak{g}(k) := \{\mathcal{P} \in \mathfrak{so}(V) \mid \mathcal{P} \cdot R_p = \mathcal{P} \cdot (\nabla R)_p = \ldots = \mathcal{P} \cdot (\nabla^k R)_p = 0\},\ k \geq 0.$$

Here $(\nabla^k R)_p$ is the value at p of the k-th covariant derivative of the curvature tensor R, and the endomorphism $\mathcal{P}$ acts on the tensor algebra of V as a derivation (see [1], p.25). By definition, $\nabla^0 R = R$. We speak of $\mathfrak{g}(k)$ as the stabiliser of the k-th covariant derivative of the Riemannian curvature tensor.

Definition 1.1. *The **Singer invariant** k_g of a locally homogeneous Riemannian manifold (M, g) is defined by*

$$k_g := min\{k \in \mathbb{N} \mid \mathfrak{g}(k) = \mathfrak{g}(k+1)\}.$$

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