A note on the torsion elements in the centralizer of a finite index subgroup

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1 Introduction

This paper is inspired by the following theorem from algebraic topology: Let Π be the fundamental group of a closed aspherical manifold and let $1 \to \Pi \to E \to F \to 1$ be any extension of Π by a finite group F, then the set of torsion elements of $C_E\Pi$ is a characteristic subgroup of E (cf. [4], [2], [3], [1],...). In the topological setting, this group of torsion elements occurs as the kernel of a properly discontinuous action.

This theorem was proved in several steps, where the first step was the case where Π was free abelian of rank k. However, although the problem could be stated in a pure algebraic way, the author could not locate any algebraic proof. E.g. to prove the (key) case where $\Pi = \mathbb{Z}^k$, one uses the specialised theory of injective toral actions, which is not easily accessible for pure algebrists.
Nevertheless, as we will show, it is not the theorem itself which depends upon the topological properties of the group Π, rather the proofs themselves. The statement of the theorem is valid in a much more general situation and is of interest from the algebraic point of view too. Therefore we would like to present a completely algebraic approach here.

2 The results

For groups G and H, we will use the following notations: C_GH is the centralizer of H in G, $Z(G)$ denotes the center of G and $\tau(G)$ equals the set of torsion elements

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