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On Prime Ideals of Lie Algebras

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Introduction. The notion of prime ideals plays an important role in the theory of associative algebras. It seems to be interesting for us to know how the corresponding notion behaves itself in Lie algebras. In this paper we shall introduce the notion of prime ideals into Lie algebras which are not necessarily finite-dimensional and investigate their properties.

We give two conditions for ideals to be prime and study the interrelations among prime, semi-prime, irreducible and maximal ideals. We also show that in a Lie algebra satisfying the maximal condition for ideals, any semi-prime ideal is an intersection of finite number of prime ideals and the unique maximal solvable ideal is equal to the intersection of all prime ideals.

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1. Let Φ be a field of arbitrary characteristic. Let L be always a Lie algebra over Φ which is not necessarily finite-dimensional. For any element x of L, $<x^L>$ is the smallest ideal of L containing x [4]. $\mathrm{Rad}_{\mathfrak{S}}(L)$ is the sum of all solvable ideals of L [6]. If L satisfies the maximal (resp. minimal) condition for ideals, we write $L \in \mathrm{Max}-\lhd$ (resp. $\mathrm{Min}-\lhd$) [5].

2. An ideal P of L is called *prime* if $[H, K] \subseteq P$ with H, K ideals of L implies $H \subseteq P$ or $K \subseteq P$.

Let L and L' be Lie algebras and let $f: L \to L'$ be a surjective homomorphism. Then it is easily seen that an ideal P of L containing $\mathrm{Ker} f$ is prime if and only if $f(P)$ is prime in L'.

Theorem 1. *Let P be an ideal of L. Then the following conditions are equivalent:*

i) *P is prime.*

ii) *If $[a, H] \subseteq P$ for $a \in L$ and an ideal H of L, then either $a \in P$ or $H \subseteq P$.*

iii) *If $[a, <b^L>] \subseteq P$ for a, $b \in L$, then either $a \in P$ or $b \in P$.*

Proof. i)$\Rightarrow$iii). For each $a \in L$,