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Divisorial Objects in Abelian Categories

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Introduction

Recently, in [3], the notion of divisorial modules was introduced in Mod(R), the category of R-modules, where R is a completely integrally closed domain. In [3], the class of all pseudo-null modules is a Serre subcategory, more precisely, a localizing subcategory of Mod(R). This fact is meaningful. In fact, if R is a commutative ring with unit, then some closure operations on the lattice of ideals of R which have the same characters as the divisorial envelope of ideals, correspond to localizing subcategories of Mod(R). Another important fact is the following: If R is noetherian, then there is a one-to-one correspondence between the class of localizing subcategories of Mod(R) and the class of subsets of Spec(R) which are stable under specialization. And if Z is a subset of Spec(R), stable under specialization, then we can define the local cohomology modules with supports in Z. Therefore, there must be some relationship between the divisorial envelopes (more generally, *$\mathscr{C}$-divisorial envelopes*, defined in § 2) of modules and the local cohomology modules. In this paper, we shall study the above problem, mostly in § 2. Since both the divisorial envelopes of R-modules and the local cohomology modules are defined functorially, we shall deal with all things in an abelian category and its localizing subcategories.

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§ 1. Weak $\mathscr{C}$-envelopes

Let $\mathscr{A}$ be an abelian category, $\mathscr{C}$ a Serre subcategory of $\mathscr{A}$. For the definitions of $\mathscr{C}$-closed objects, $\mathscr{C}$-isomorphisms and $\mathscr{C}$-envelopes, we shall refer to [1]. Also, we shall assume basic properties of them (see [1] or [2]). For the purpose of convenience, we say that an object L is *$\mathscr{C}$-pure* if L has no $\mathscr{C}$-subobjects. The following lemma is an easy consequence of this definition.

LEMMA 1.1. *An object L is $\mathscr{C}$-pure if and only if, for every $\mathscr{C}$-isomorphism α: $M \to N$, $\mathrm{Hom}(N, L) \to \mathrm{Hom}(M, L)$ is injective.*

PROOF. (Necessity) Let f: $N \to L$ be a morphism. Suppose $\alpha f = 0$, then there is an epimorphism $\mathrm{Coker}(\alpha) \to \mathrm{Im}(f)$. Hence $\mathrm{Im}(f)$ is a $\mathscr{C}$-subobject of