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# *On the Decomposition of Coalgebras*

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## Introduction

It is well known that a cocommutative coalgebra is decomposed into the direct sum of its irreducible components ([3], Theorem 8. 0. 5). This theorem is based on the fact that a commutative artinian ring is uniquely decomposed into a direct product of local rings ([4], p.205). As for the non-commutative case, we know that an artinian ring is the direct sum of its blocks which are uniquely determined and indecomposable as two-sided ideals. A block is the sum of principal indecomposable modules which are linked to each other (e.g., [1], p.171).

The purpose of this paper is to give a criterion for a coalgebra to be indecomposable. Here, a coalgebra is indecomposable provided it cannot be decomposed into a direct sum of two non-zero subcoalgebras.

Since the Brauer's theory stated above is constructed by using one-sided ideals, it seems not to be useful for a theory of coalgebras. Thus we consider the decomposition of rings from a slightly different viewpoint.

Let $C$ be a coalgebra over a field. Then we shall prove the following result: A necessary and sufficient condition for $C$ to be indecomposable is that for any simple subcoalgebras $S$ and $S'$ there exists a sequence

$$S = S_1, S_2,\dots, S_r = S'$$

of simple subcoalgebras such that

$$S_i \wedge S_{i+1} \neq S_{i+1} \wedge S_i \qquad \text{for} \quad i = 1,\dots, r-1.$$

The proof is divided into two parts. First, in Section 2, we reduce this problem to the finite-dimensional case, where coalgebras can be completely turned into algebras. Secondly, in Section 3, we prove the paraphrased assertion.

All notations and terminology we will use are the same as in [3].

## 1. Notations, Definitions and Main Theorem

Throughout this paper all coalgebras and algebras are over a fixed field $k$. Let $C$ be a coalgebra. We denote by $\mathfrak{S} = \mathfrak{S}(C)$ the set of all simple subcoalgebras of $C$.