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Fibred Riemannian spaces with contact structure

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Introduction

Theory of foliated Riemannian manifolds has been studied by many authors in the name of fibred spaces ([3]) or foliated manifolds ([8], [14]) by use of Riemannian submersions ([7]). K. Ogiue ([5], [6]) studied relations between an almost contact structure and an almost complex structure induced in the base space of fibering. On the other hand, A. Morimoto ([4]) defined an almost complex structure in the product of two almost contact spaces. Y. Tashiro and the present author ([12]) have recently induced an almost complex structure in the total space of a fibred Riemannian space, the base space and each fibre of which are almost contact, and investigated relations among their structures.

The purpose of this paper is to study fibred spaces with almost contact metric structure induced from the base space with almost complex structure and each fibre with almost contact metric structure of general dimensions. A typical example of these is the Hopf fibering π: $S^{4n+3}(1) \to QP(n)$ with totally geodesic fibre S^3 (cf. [2], [11]).

In §1, we shall summarize fundamental properties and known results of the fibred Riemannian space. We shall induce, in §§2 and 3, an almost contact metric structure on the total space by use of the almost complex structure on the base space and almost contact metric structure on each fibre and discuss relations of them. §4 is devoted to the study of space form and we shall prove that the base space is locally Euclidean if the total space is Sasakian space form with conformal fibres. An example having this property will be given. In the last section, we shall investigate relations between the integrability of the almost complex structure and the normality of the induced almost contact metric structure on the total space by use of Nijenhuis tensors.

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§1. Fibred Riemannian spaces

Let $\{\tilde{M}, M, \tilde{g}, \pi\}$ be a fibred Riemannian space, that is, $\{\tilde{M}, \tilde{g}\}$ is an m-dimensional total space with Riemannian metric $\tilde{g}$, M an n-dimensional base space, π: $\tilde{M} \to M$ the projection with maximum rank n. The fibre passing through a point