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Fibred Sasakian spaces with vanishing contact Bochner curvature tensor

Byung Hak KIM

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Introduction

There have been many attempts to clarify geometric meanings of Bochner curvature since S. Bochner [3] introduced it as a Kaehlerian analogue of conformal curvature in 1949. S. Tachibana [12] gave the expression of Bochner curvature tensor in real form, M. Matsumoto and S. Tanno [10] proved that a Kaehlerian space with vanishing Bochner curvature tensor and of constant scalar curvature is a complex space form or a locally product of two complex space forms of constant holomorphic sectional curvature c (≥ 0) and $-c$. Y. Kubo [8], I. Hasegawa and T. Nakane [5] obtained necessary conditions for a Kaehler manifold with vanishing Bochner curvature tensor to be a complex space form.

On the other hand, M. Matsumoto and G. Chūman [9] defined the contact Bochner (briefly, C-Bochner) curvature tensor in a Sasakian space and studied its properties. A Sasakian space form is a space with vanishing C-Bochner curvature tensor.

In this paper, we discuss properties of fibred Sasakian spaces with vanishing C-Bochner curvature tensor and construct an example of Sasakian space with vanishing C-Bochner curvature tensor which is not a Sasakian space form. As to notations and terminologies, we refer to the previous papers [7, 13].

Throughout this paper, the ranges of indices are as follows:

$$\begin{aligned} A, B, C, D, E &= 1, 2, \ldots, m, \\ h, i, j, k, l &= 1, 2, \ldots, m, \\ a, b, c, d, e &= 1, 2, \ldots, n, \\ \alpha, \beta, \gamma, \delta, \varepsilon &= n+1, \ldots, n+p = m. \end{aligned}$$

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§1. Preliminaries

Let $\{\tilde{M}, M, \tilde{g}, \pi\}$ be a fibred Riemannian space, that is, $\{\tilde{M}, \tilde{g}\}$ is an m-dimensional total space with projectable Riemannian metric $\tilde{g}$, M an