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Pettis integrability and the equality of the norms of the weak* integral and the Dunford integral

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Abstract It is shown that if (Ω, Σ, μ) is a finite measure space and X is a Banach space then X^* has the μ-Pettis Integral Property if and only if

$$\|(weak^*) - \int_\Omega f d\mu\| = \|(Dunford) - \int_\Omega f d\mu\|$$

for every bounded weakly measurable function $f\colon \Omega \to X^*$.

A negative answer to a question of E. Bator is also given.

1. Introduction

Let (Ω, Σ, μ) be a finite measure space. For a Banach space X we denote by $bwm(\mu; X)$ the space of all bounded and weakly measurable X-valued functions defined on Ω. X^* denotes the dual of X. B is the unit ball of X.

It is well known that if $f \in bwm(\mu; X^*)$ then for every $E \in \Sigma$ there exists $x_E^* \in X^*$ such that, for every $x \in X$,

$$x_E^*(x) = \int_E x f d\mu$$

and, for every $E \in \Sigma$ there exists $x_E^{***} \in X^{***}$ such that, for every $x^{**} \in X^{**}$,

$$x_E^{***}(x^{**}) = \int_E x^{**} f d\mu.$$

x_E^* is called the *weak* integral* of f over E, denoted by $w^* - \int_E f d\mu$, and x_E^{***} is called the *Dunford integral* of f over E, denoted by $D - \int_E f d\mu$. f is said to be *Pettis integrable* if $D - \int_E f d\mu \in X^*$ for all $E \in \Sigma$.

X is said to have the *μ-Pettis Integral Property* (μ-PIP) if every $f \in bwm(\mu; X)$ is Pettis integrable. More information on Pettis integral and μ-PIP can be found in [3].