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A note on the Selberg zeta function for compact quotients of hyperbolic spaces

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0. Introduction

In the previous paper [18], we have worked out a new method of the analytic continuation of the logarithmic derivative of Selberg's zeta function for the compact Riemannian surfaces X. In the present note, we consider the generalization of those results when X is a certain compact quotient of hyperbolic space $\tilde{X}$.

Now, let G be a connected component of the isometric transformation group of $\tilde{X}$. Let K be a maximal compact subgroup of G. Then $\tilde{X}$ is realized by G/K, a noncompact symmetric space of rank one. Let Γ be a discrete torsion-free subgroup of G such that $\Gamma\backslash G$ is compact. Thus our manifold X treating throughout this note is given by $\Gamma\backslash G/K$ for some such Γ. Let T be a finite-dimensional unitary representation of Γ, and let χ be its character. In the monumental work [15], A. Selberg constructed a function of complex variable $Z_\Gamma(s, \chi)$, which is called Selberg's zeta function, and showed how the location and the order of its zeroes gave us information about the spectrum of the Laplacian for X on the one hand and about the topology of X on the other hand. Furthermore, in the famous paper [4], R. Gangolli extended these results in the more general situation, that is to say, in the present case.

The zeta function of Selberg's type is given by the infinite product which converges absolutely for some half plane, say $\Re s > 2\rho_0$, where $\Re s$ stands for the real part of $s \in \boldsymbol{C}$. Here, the number ρ_0 is a positive real one depending only on $\tilde{X}$ and the product is taken over all primitive conjugacy classes in Γ and a certain semi-lattice of roots with respect to the Cartan subgroup of G. Roughly speaking, this zeta function has the following properties:

(A) $Z_\Gamma(s, \chi)$ is holomorphic in a half plane $\Re s > 2\rho_0$ and has a meromorphic continuation to the whole complex plane.

(B) $Z_\Gamma(s, \chi)$ satisfies the functional equation

$$Z_\Gamma(2\rho_0 - s, \chi) = \left\{\exp(\chi(1)\mathscr{V}(X)\int_0^{s-\rho_0} \mu(iz)dz\right\} Z_\Gamma(s, \chi).$$

Here, $\mathscr{V}(X)$ is the volume of X in a suitable normalization and $\mu(z)dz$ is