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Existence of non-constant stable equilibria in competition-diffusion equations

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1. Introduction

The study of coexistence problem of competing species is one of the main topics in mathematical ecology. In this paper we study the relation between the coexistence of two competing species and the domain shape of their habitat.

The model system which we use here is the following Lotka-Volterra type reaction-diffusion equation:

$$(1.1)\qquad \begin{cases} u_t = d_u \Delta u + u(a_u - b_u u - c_u v), \\ v_t = d_v \Delta v + v(a_v - b_v u - c_v v), \qquad x \in \Omega,\ t > 0, \\ \dfrac{\partial}{\partial \nu} u(t, x) = 0 = \dfrac{\partial}{\partial \nu} v(t, x), \qquad x \in \partial\Omega,\ t > 0, \\ u(0, x) \geq 0,\ v(0, x) \geq 0, \qquad x \in \bar{\Omega}, \end{cases}$$

where Δ is the Laplace operator, Ω a bounded domain in $\mathbf{R}^{n+1}$, u and v the population densities of the two competing species. The constants a_u and a_v are the intrinsic growth rates, b_u, c_v and b_v, c_u the coefficients of intraspecific and interspecific competition, respectively, d_u and d_v the diffusion rates. We assume that all these constants are positive.

By a suitable normalization, we can rewrite (1.1) as

$$(1.2)\qquad \begin{cases} u_t = \varepsilon^2 \Delta u + u(1 - u - cv), \\ v_t = \varepsilon^2 d \Delta v + v(a - bu - v), \qquad x \in \Omega,\ t > 0, \\ \dfrac{\partial}{\partial \nu} u\,(t, x) = 0 = \dfrac{\partial}{\partial \nu} v(t, x), \qquad x \in \partial\Omega,\ t > 0, \\ u(0, x) \geq 0,\ v(0, x) \geq 0, \qquad x \in \bar{\Omega}, \end{cases}$$

where a, b, c, d, ε are positive. The following result is well-known (for example, see de Mottoni [7]):

(I) If $a < \min\{b, 1/c\}$, then $\lim_{t \to \infty}(u(t, x), v(t, x)) = (1, 0)$.