HIROSHIMA MATH. J. **28** (1998), 261–308

## Bifurcation theory for semilinear elliptic boundary value problems

## Kazuaki TAIRA

(Received February 12, 1997)

**ABSTRACT.** This expository paper is devoted to static bifurcation theory for a class of *degenerate* boundary value problems for semilinear second-order elliptic differential operators stimulated by a problem of chemical kinetics. Our approach is distinguished by the extensive use of the ideas and techniques characteristic of the recent developments in the theory of partial differential equations.

## Table of contents

- 0. Introduction and results
- 1. Functional analytic preliminaries
  - 1.1 Theory of positive mappings in ordered Banach spaces
  - 1.2 Local bifurcation theory
- 2. Proof of Theorem 0
  - 2.1 Existence and uniqueness theorem for problem (0.1)
  - 2.2 Selfadjointness of the operator  $\mathfrak{A}$
  - 2.3 Positivity of the resolvent K
  - 2.4 End of Proof of Theorem 0
- 3. Proof of Theorem 1
- 4. Proof of Theorems 2 and 3
  - 4.1 Proof of Theorem 2
  - 4.2 Proof of Theorem 3
- 5. Proof of Theorem 4
- 6. Proof of Theorem 5
- 7. Appendix: The maximum principle References

## 0. Introduction and results

Let D be a bounded domain of Euclidean space  $\mathbb{R}^N$  with smooth boundary  $\partial D$ ; its closure  $\overline{D} = D \cup \partial D$  is an N-dimensional, compact smooth manifold

<sup>1991</sup> Mathematics Subject Classification. Primary 35B32, 35J65; Secondary 35P15, 35P30.

Key words and phrases. Bifurcation, simple eigenvalue, super-subsolution method, semilinear elliptic problem.