Carleson inequalities in classes of derivatives of harmonic Bergman functions with $0 < p \leq 1$

Dedicated to Professor Jyunji Inoue on his sixtieth birthday

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ABSTRACT. We give a necessary and sufficient condition for a positive measure μ on the upper half-space of $\mathbf{R}^n$ to satisfy the inequalities

$$\left(\int |D^\alpha u|^q d\mu\right)^{1/q} \leq C\left(\int |D_y^m u|^p y^r dV\right)^{1/p}$$

for all u in a subclass of a harmonic Bergman space when $0 < p \leq 1$ and $p \leq q$, where D_y denotes the partial differentiation operator with respect to the last coordinate y. We also show that the Bergman norm is comparable to derivative norms and harmonic conjugation is bounded on the harmonic Bergman space b^p when $0 < p \leq 1$.

1. Introduction

Let H be the upper half-space of the n-dimensional Euclidean space $\mathbf{R}^n (n \geq 2)$, that is, $H = \{z = (x, y) \in \mathbf{R}^n; y > 0\}$, where we have written a point $z \in \mathbf{R}^n$ as $z = (x, y)$ with $x = (x_1, \ldots, x_{n-1}) \in \mathbf{R}^{n-1}$ and $y \in \mathbf{R}$. For $0 < p < \infty$, let $b^p = b^p(H, dV)$ be the class of all harmonic functions u on H such that

$$\|u\|_p = \left(\int_H |u|^p dV\right)^{1/p} < \infty$$

where dV denotes the Lebesgue volume measure on H. The class b^p is called the harmonic Bergman space. Recently, properties of functions in the harmonic Bergman space b^p for $1 \leq p < \infty$ have been studied by Ramey and Yi [9], and several important results have been given. Our aim is to investigate properties in the harmonic Bergman space b^p when $p \leq 1$.

In this paper, we study conditions on a σ-finite positive Borel measure μ on

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