Oscillation criteria for first order nonlinear delay differential equations

Jianchu Jiang

Abstract. In this paper, some new oscillation criteria are obtained for the first order nonlinear delay differential equation

$$x'(t) + p(t)f(x(t - \tau_1), \dots, x(t - \tau_m)) = 0$$

and the corresponding advanced differential equation. Our results improve the known results in the literature. And an example is given to demonstrate the advantage of our results.

1. Introduction

The oscillatory behavior of differential equations with deviating arguments has been studied by many authors. For some contributions in this area see the papers [1–11].

Consider the first order delay differential equation

$$x'(t) + p(t)f(x(t - \tau_1), \dots, x(t - \tau_m)) = 0, \qquad t \ge t_0 \qquad (1)$$

and the advanced differential equation

$$x'(t) - p(t)f(x(t + \tau_1), \dots, x(t + \tau_m)) = 0, \qquad t \ge t_0, \qquad (2)$$

where $p(t) \ge 0$ is a continuous function, $0 < \tau_1 \le \tau_2 \le \cdots \le \tau_m$, and the function f satisfies the following conditions:

(H$_1$). f is continuous on $\mathbf{R}^m$ and such that

$$y_i > 0 \qquad \text{for } i = 1, \dots, m \Rightarrow f(y_1, \dots, y_m) > 0$$

and

$$y_i < 0 \qquad \text{for } i = 1, \dots, m \Rightarrow f(y_1, \dots, y_m) < 0;$$