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The Radon transform on an exceptional flag manifold

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Abstract. We shall give a description of certain Radon transforms between generalized flag manifolds for the simple algebraic group of type (E_7) from the view point of the D-module theory.

1. Introduction

Let G be a connected simple algebraic group over the complex number field $\mathbf{C}$, and let P and Q be parabolic subgroups of G containing the same Borel subgroup. Set $X = G/P$, $Y = G/Q$, $Z = G/(P \cap Q)$ and consider the correspondence:

$$Y \overset{q}{\leftarrow} Z \overset{p}{\rightarrow} X.$$

Assume that an invertible $\mathcal{O}_X$-module $\mathscr{L}$ and an invertible $\mathcal{O}_Y$-module $\mathscr{M}$ satisfy $q^*\mathscr{M} \otimes_{\mathcal{O}_Z} \Omega_{Z/X}^{\otimes -1} \cong p^*\mathscr{L}$, where $\Omega_{Z/X}$ denotes the sheaf of relative differential forms of maximal degree along the fibers of p. Define the sheaves of twisted differential operators $D_{X,\mathscr{L}}$ and $D_{Y,\mathscr{M}}$ on X and Y by

$$D_{X,\mathscr{L}} = \mathscr{L} \otimes_{\mathcal{O}_X} D_X \otimes_{\mathcal{O}_X} \mathscr{L}^{\otimes -1}, \qquad D_{Y,\mathscr{M}} = \mathscr{M} \otimes_{\mathcal{O}_Y} D_Y \otimes_{\mathcal{O}_Y} \mathscr{M}^{\otimes -1},$$

respectively. For a $D_{Y,\mathscr{M}}$-module N set

$$R(N) = \int_p (\Omega_{Z/X}^{\otimes -1} \otimes_{\mathcal{O}_Z} q^*N).$$

It is a complex of $D_{X,\mathscr{L}}$-modules, called the Radon transform of N. This integral transform plays important roles in some aspects of the representation theory as well as in the theory of certain hypergeometric type differential equations (see Oshima [6], Tanisaki [8], and their references). In the most fundamental case where $N = D_{Y,\mathscr{M}}$ we have a canonical morphism

$$\Phi : D_{X,\mathscr{L}} \to H^0(R(D_{Y,\mathscr{M}})).$$

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