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Extensions and the irreducibilities of the induced characters of cyclic p-groups

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ABSTRACT. Let ϕ be a faithful irreducible character of the cyclic group C_n of order p^n, where p is an odd prime. We study the p-group G containing C_n such that the induced character ϕ^G is also irreducible. The purpose of this paper is to determine the subgroups $N_G(N_G(C_n))$ and $N_G(N_G(N_G(C_n)))$ of G in the case when $[N_G(C_n); C_n] = p$.

1. Introduction

Let G be a finite group. We denote by $\mathrm{Irr}(G)$ the set of complex irreducible characters of G and by $\mathrm{FIrr}(G)$ $(\subset \mathrm{Irr}(G))$ the set of faithful irreducible characters of G.

Let p be a prime. For a non-negative integer n, we denote by C_n the cyclic group of order p^n. A finite group G is called an M-group, if every $\phi \in \mathrm{Irr}(G)$ is induced from a linear character of a subgroup of G.

It is well-known that every nilpotent group is an M-group. Hence, when G is a p-group, for any $\chi \in \mathrm{Irr}(G)$, there exists a subgroup H of G and a linear character ϕ of H such that $\phi^G = \chi$. If we set $N = \mathrm{Ker}\,\phi$, then $N \lhd H$ and ϕ is a faithful irreducible character of $H/N \cong C_n$, for some non-negative integer n. In this paper, we will consider the case when $N = 1$, that is, ϕ is a faithful linear character of $H \cong C_n$.

We consider the following:

PROBLEM 1. *Let p be an odd prime, and ϕ be a faithful irreducible character of C_n. Determine the p-group G such that $C_n \subset G$ and the induced character ϕ^G is also irreducible.*

Since all the faithful irreducible characters of C_n are algebraically conjugate to each other, the irreducibility of ϕ^G $(\phi \in \mathrm{FIrr}(C_n))$ is independent of the choice of ϕ, and depends only on n.

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