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On the reflexivity of certain fibered 3-knots

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§1. Introduction

Given $A \in GL(n, \boldsymbol{Z})$, $\Lambda^i A$ denotes its i-fold exterior product with itself. Let A satisfy the CS condition: $n \geq 3$, $\det A = 1$ and $\det(\Lambda^i A - I) = \pm 1$ for $1 \leq i \leq [n/2]$. The mapping torus of a diffeomorphism on a torus T^n induced by A is a homology $S^1 \times S^n$ and has a unique loop up to isotopy whose conjugates generate the fundamental group ([2]). Taking surgery on the loop we get a pair of $(n-1)$-knots in homotopy $(n+1)$-spheres according to the two framings. If these knots are equivalent, then the knot is called reflexive. The reflexive knot is determined by its exterior. The criterion for the reflexivity is given by Cappell-Shaneson.

PROPOSITION 1. (Cappell-Shaneson [2]) *Let $A \in GL(n, \boldsymbol{Z})$ satisfy the CS condition. Then the associated knot is reflexive if and only if there is a matrix $B \in GL(n, \boldsymbol{Z})$ such that $AB = BA$ and the restriction of B to the negative eigenspace of A has negative determinant.*

The purpose of this paper is to prove the following theorem by using Cappell-Shaneson criterion and extending the technique of Hillman-Wilson [3].

THEOREM 2. *Let $A \in GL(4, \boldsymbol{Z})$ satisfy the CS condition and* $\det(A - I) = 1$. *Then the characteristic polynomial $f_A(t)$ of A is either* (1) $t^4 + at^3 - 2(a+1)t^2 + (a+1)t + 1$ or (1)$'$ $t^4 + at^3 - 2at^2 + (a-1)t + 1$ *where $a = -$ trace (A). The associated knot is reflexive if and only if A is of type* (1) *and $a = 0$, or A is of type* (1)$'$ *and $a = 1$.*

REMARK 1. The non-reflexivity of the case (1) with $a \leq -1$ is noted in [3].

REMARK 2. If $\det(A - I) = -1$, then $f_A(t)$ is either (2) $t^4 + at^3 - 2(a+2)t^2 + (a+1)t + 1$ or (2)$'$ $t^4 + at^3 - 2(a+1)t^2 + (a-1)t + 1$. The reflexivity of the knot in this case is not completely determined yet.

Before closing the introduction we note that the characteristic polynomial of $A \in GL(n, \boldsymbol{Z})$ satisfying the CS condition is irreducible. In fact, the CS condition implies $\det(\Lambda^i A - I) = \pm 1$ for $1 \leq i \leq n-1$. By Newman [5, p. 50] any square matrix A over $\boldsymbol{Z}$ is similar to some block triangular matrix