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Semi-Infinite Programs and Conditional Gauss Variational Problems

Maretsugu Yamasaki

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§1. Introduction

Let X and Y be real linear spaces which are in duality with respect to a bilinear functional $((\ ,\ ))_1$ and let Z and W be real linear spaces which are in duality with respect to a bilinear functional $((\ ,\ ))_2$. Denote by $w(X, Y)$ the weak topology on X. An infinite linear program for these paired spaces is a quintuple (A, P, Q, y_0, z_0). In this quintuple, A is a linear transformation from X into Z which is $w(X, Y)-w(Z, W)$ continuous, P is a convex cone in X which is $w(X, Y)$-closed, Q is a convex cone in Z which is $w(Z, W)$-closed, $y_0 \in Y$ and $z_0 \in Z$ are fixed elements. One of the basic problems in the theory of linear programming is to determine the value M of the program defined by

$$M = \inf\{((x, y_0))_1;\ x \in S\},$$

where

$$S = \{x \in P;\ Ax - z_0 \in Q\}.$$

In this paper, we use the convention that the infimum and the supremum on the empty set ϕ are equal to $+\infty$ and $-\infty$ respectively.

The dual problem is to determine the value M^* defined by

$$M^* = \sup\{((z_0, w))_2;\ w \in S^*\},$$

where

$$S^* = \{w \in Q^+;\ y_0 - A^*w \in P^+\}.$$

Here A^* denotes the adjoint transformation of A, i.e., A^* is the linear transformation from W into Y which is $w(W, Z)-w(Y, X)$ continuous and satisfies the relation

$$((Ax, w))_2 = ((x, A^*w))_1$$

for all $x \in X$ and $w \in W$ and P^+ and Q^+ are defined by

$$P^+ = \{y \in Y;\ ((x, y))_1 \geqq 0 \text{ for all } x \in P\},$$