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Oscillatory solutions of functional differential equations generated by deviation of arguments of mixed type

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1. Introduction

In this paper we consider the higher order functional differential equation of the form

(E) $x^{(n)}(t)+F(t, x(t), x(g_1(t)),\dots, x(g_N(t)),\dots, x^{(n-1)}(t),\dots, x^{(n-1)}(g_N(t)))=0$

and its particular cases

(A) $$x^{(n)}(t) - \sum_{h=1}^{N} p_h(t)f_h(x(g_h(t))) = 0,$$

(B) $$x^{(n)}(t) + \sum_{h=1}^{N} p_h(t)f_h(x(g_h(t))) = 0,$$

where the deviating arguments $g_h(t)$ are of general type.

It is assumed that the function $F(t, u_0,\dots, u_N, u_0^{(1)},\dots, u_N^{(n-1)})$ satisfies either the condition

(1) $$F(t, u_0,\dots, u_N, u_0^{(1)},\dots, u_N^{(1)},\dots, u_N^{(n-1)})u_0 \geq 0$$

or the condition

(2) $$F(t, u_0,\dots, u_N, u_0^{(1)},\dots, u_N^{(1)},\dots, u_N^{(n-1)})u_0 \leq 0$$

in the domain

$$\Omega = \{(t, u_l^{(q)})\colon t\in[a, \infty), u_0u_l > 0, 0\leq l\leq N, 0\leq q\leq n-1\}.$$

By a *proper solution* of equation (E) we mean a function $x\in C^n[[T_x, \infty), R]$ which satisfies (E) for all sufficiently large t and $\sup\{|x(t)|\colon t\geq T\}>0$ for any $T\geq T_x$. We make the standing hypothesis that equation (E) does possess proper solutions. A proper solution of (E) is called *oscillatory* if it has arbitrarily large zeros; otherwise the solution is called *nonoscillatory*.

When equation (E) is ordinary ($g_i(t)\equiv t$, $1\leq i\leq N$), the oscillatory properties of (E) satisfying (1) are essentially different from those of (E) satisfying (2). In case (1) holds, the "property A" is typical for equation (E): if n is even, then all proper solutions are oscillatory, and if n is odd, then every proper solution is either oscillatory or monotonically tending to zero as $t\to\infty$. On the other hand, when (2) holds the "property B" is typical: if n is even, then every solution is either oscillatory or else tending monotonically to infinity or to zero as $t\to\infty$, and if n