HIROSHIMA MATH. J.
14 (1984), 401–406

Boundary limit of discrete Dirichlet potentials

Takashi KAYANO and Maretsugu YAMASAKI

(Received January 11, 1984)

§1. Introduction

Let X be a countable set of nodes, Y be a countable set of arcs, K be the node-arc incidence function and r be a strictly positive real function on Y. The quartet $N=\{X, Y, K, r\}$ is called an infinite network if the graph $\{X, Y, K\}$ is connected, locally finite and has no self-loop. For notation and terminology, we mainly follow [4], [6] and [7].

Let $L(X)$ be the set of all real functions on X and $L_0(X)$ be the set of all $u \in L(X)$ with finite support. For $u \in L(X)$, its Dirichlet integral $D_p(u)$ of order p $(1<p\leq\infty)$ is defined by

$$D_p(u) = \sum_{y\in Y} r(y)^{1-p}|\sum_{x\in X} K(x, y)u(x)|^p \quad (1 < p < \infty),$$

$$D_\infty(u) = \sup_{y\in Y} r(y)^{-1}|\sum_{x\in X} K(x, y)u(x)|.$$

Denote by $\boldsymbol{D}^{(p)}(N)$ the set of all $u \in L(X)$ with finite Dirichlet integral of order p. It is easily seen that $\boldsymbol{D}^{(p)}(N)$ is a Banach space with the norm $\|u\|_p=[D_p(u)+|u(b)|^p]^{1/p}$ $(1<p<\infty)$ and $\|u\|_\infty=D_\infty(u)+|u(b)|(b\in X)$.

Denote by $\boldsymbol{D}_0^{(p)}(N)$ the closure of $L_0(X)$ in $\boldsymbol{D}^{(p)}(N)$ with respect to the norm $\|u\|_p$. This $\boldsymbol{D}_0^{(p)}(N)$ is determined independently of the choice of b. As in the continuous potential theory, we may call an element of $\boldsymbol{D}_0^{(p)}(N)$ a (discrete) Dirichlet potential of order p.

A typical Dirichlet potential of order 2 is the Green function g_a of N with pole at a (cf. [1]). This is defined by the conditions: $g_a \in \boldsymbol{D}_0^{(2)}(N)$ and $\Delta g_a(x)=-\varepsilon_a(x)$ on X, where Δ is the discrete Laplace operator and ε_a is the characteristic function of the set $\{a\}$. It was shown in [7] that the Green function g_a exists if and only if N is of hyperbolic type of order 2, or equivalently $\boldsymbol{D}^{(2)}(N)\neq\boldsymbol{D}_0^{(2)}(N)$.

In the case where $\{X, Y, K\}$ is the lattice domain in the 3-dimensional Euclidean space and $r=1$, Duffin [2] showed by means of Fourier analysis that g_a vanishes at the ideal boundary ∞ of N. In the general case, $g_a(x)$ does not always have limit 0 as x tends to the ideal boundary ∞ of N along a path from a to ∞.

In this paper, we are concerned with the boundary behavior of Dirichlet potentials of order p. Namely, we aim to show that for every $u \in \boldsymbol{D}_0^{(p)}(N)$ the set of all paths along which $u(x)$ does not have limit 0 as x tends to the ideal boundary ∞ of N is a small set in some sense. As in the continuous case (cf. [5]), we use